

## Supplementary File

## Tree-based differential testing using inferential uncertainty for RNA-Seq

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## S1 Methods

### S1.1 Range-factorized equivalence class

12 An equivalence class denotes an association from a set of transcripts to a set of reads, that are mapped to all the  
 13 transcripts in that set. A **range-factorized** equivalence class in addition also encodes the mapping quality, with a single  
 14 class constituting a set of pairs  $(t_i, w_i)$  rather than just a set of  $t_i$ , where  $t_i$  denotes the transcript and  $w_i$  represents the  
 15 average conditional probability with which the fragments in the equivalence class arose from that transcript.

## 16 S1.2 TreeTerminus

17 For a given RNA-Seq experiment consisting of  $M$  samples, **TreeTerminus** [6] outputs a forest of  $K$  trees  $\mathcal{T} = \{T_1, T_2, \dots, T_K\}$ ,  
 18 that summarize the abundance uncertainty structure across all the  $M$  samples. The leaves of the individual trees comprise  
 19 the set of quantified transcripts and each internal node represents an aggregation of the set of transcripts belonging to  
 20 the subtree rooted at it, with no two trees having an overlapping set of transcripts/leaves. The input to **TreeTerminus** is  
 21 the **salmon** [7] quantification estimates,  $L$  inferential replicates, and range-factorized equivalence classes [5] corresponding  
 22 to each RNA-Seq sample  $m$ ,  $m \in \{1, \dots, M\}$ . The  $L$  inferential replicates are produced either through Gibbs sampling or  
 23 bootstrap sampling and are denoted by  $\mathcal{I}_{mi} = \{I_{mi_1}, I_{mi_2}, \dots, I_{mi_L}\}$ , where  $I_{mi_l}$  represents the counts of the transcript  $i$

24 for sample  $m$  at the  $l^{th}$  Gibbs/bootstrap iteration. The inferential replicate counts for an inner node are found by adding  
 25 inferential replicate counts of each constituent transcript individually. The tree represents the order in which the different  
 26 transcripts are aggregated into transcript groups starting from the leaf nodes and encoding different resolution layers for  
 27 interpretation of the abundance of transcriptional groups, with uncertainty generally decreasing as one ascends the tree  
 28 from the leaves. The uncertainty for any node (leaf or inner node)  $n$  for a given sample  $m$  is estimated using the metric  
 29 `infRV` defined in [3], over  $\mathcal{I}_{mn}$  as:

$$\text{infRV}_{mn} = \frac{\max(\sigma_{I_{mn}}^2 - \mu_{I_{mn}}, 0)}{\mu_{I_{mn}} + pc} + d \quad (1)$$

30 where  $\mu_{I_{mn}}, \sigma_{I_{mn}}$  are the mean, variance over the  $L$  inferential replicates for a sample  $m$  and node  $n$ ,  $pc$  is a pseudocount  
 31 (with a default value of 5) and  $d$  is a small global shift (with a default value of 0.01). The nodes situated at the lower  
 32 heights in a branch in the tree usually represent the set of nodes for which large reduction in `infRV` was observed compared  
 33 to its underlying children nodes. For most nodes in the tree, the underlying transcripts belong to the same gene, due to  
 34 large sequence overlap between them which is a driving factor behind uncertainty. However, the transcripts in a node  
 35 can also map to different genes, as there can be overlapping sequence regions between different genes as well, and also  
 36 sequence similarity between genes belonging to a gene family. While a reduction in `infRV` governs how transcripts are  
 37 aggregated into nodes, it does not mean that all the underlying transcripts will have similar strength and direction of  
 38 differential signal between the conditions of interest.

39 **Unified Tree** - Once the forest from `TreeTerminus` is obtained, a unified tree is constructed. For the sake of simplicity,  
 40 let  $\mathcal{T}$  denote the unified tree. The unified tree is constructed using the R package `beaveR`(<https://github.com/NPSDC/beaveR>). This tree is constructed by first creating a new root node and assigning all trees in the forest and the remaining  
 41 transcripts in the transcriptome not covered by the trees as children of this root node.  
 42

### 43 S1.3 Median-ratio scaled counts for the nodes in the tree

44 Using the formulation from [3], let  $Y^0$  denote the counts matrix obtained for Salmon for the transcript set containing  $M$   
 45 samples  $1, \dots, m$  and  $P$  transcripts  $1, \dots, p$ , with  $Y_{ji}^0$  representing the counts for transcript  $j$  in sample  $i$ . Let the matrix  
 46  $Y^{\mathcal{T}0}$  denote the counts obtained for all the nodes in the tree  $\mathcal{T}$  that has  $P$  leaf nodes, where for an internal node  $n$ ,  
 47  $Y_{ni}^{\mathcal{T}0} = \sum_{d=1}^{|\Lambda(n)|} Y_{t_d}^0, \forall t_d \in \Lambda(n)$ , where  $\Lambda(n)$  denote the indexes of the descendant transcripts of node  $n$ . The counts  $Y_{ji}^{\mathcal{T}}$  are  
 48 divided by a bias length term  $b_{ji}$ , accounting for the length w.r.t other transcripts:

$$b_{ji} = \frac{l_{ji}}{(\prod_{i=1}^m l_{ji})^{\frac{1}{m}}}$$

49 Then we divide the counts by  $b_{ji}$  as

$$Y_{ji}^{\mathcal{T}*} = \frac{Y_{ji}^{\mathcal{T}0}}{b_{ji}}$$

50 The counts are then scaled to the geometric mean of sequencing depth as

$$Y_{ji}^{\mathcal{T}**} = \frac{Y_{ji}^{\mathcal{T}*}}{\sum_{j=1}^P Y_{ji}^{\mathcal{T}*}} \times \left( \prod_{i=1}^m \sum_{j=1}^P Y_{ji}^{\mathcal{T}0} \right)^{\frac{1}{m}}$$

51 For each sample  $i$ , a median-ratio size factor is computed as

$$s_i = \text{median}_j^P \frac{Y_{ji}^{\mathcal{T}**}}{\left( \prod_{k=1}^m Y_{jk}^{\mathcal{T}**} \right)^{\frac{1}{m}}}$$

52 We compute the size factor over only the leaf nodes. The final normalized counts are then computed as:

$$Y_{ji}^{\mathcal{T}} = \frac{Y_{ji}^{\mathcal{T}**}}{s_i}$$

53 The lengths of the inner nodes are computed using the strategy employed by `summarizeToGene` function in the `R`  
54 package `tximport` [4]. The length of an inner node  $n$  for sample  $i$  is computed as:

$$l_{ni}^{\mathcal{T}} = \frac{\sum_{d=1}^{|\Lambda(n)|} l_{t_d i} tpm_{t_d i}}{\sum_{d=1}^{|\Lambda(n)|} tpm_{t_d i}}, \forall t_d \in \Lambda(n)$$

55 Here  $tpm$  refers to transcripts per million estimates that are provided by Salmon.

56 **S1.4 Distance between nodes**

57 The distance between the set of nodes  $N_d$  and  $N_p$  is computed as :

$$\begin{aligned} \mathcal{D}(N_d, N_p) &= \frac{dist(N_d, N_p) + dist(N_p, N_d)}{2}, \\ dist(N_a, N_b) &= \frac{\sum_{i=1}^{|N_a|} d(N_{ai})}{\|N_a\|}, \\ d(N_{ai}) &= \begin{cases} 0 & \text{if } N_{ai} \in N_b \\ Path\_length(N_{ai}, N_{bk}) & \text{if } N_{bk} \in N_b \text{ and } N_{bk} \text{ is either an ancestor} \\ & \text{or descendant of } N_{ai} \\ Path\_length(N_{ai}, \text{root}) + 1 & \text{if } N_{ai} \notin N_b \text{ and no ancestor or} \\ & \text{descendant of } N_{ai} \text{ exists in } N_b \end{cases} \end{aligned}$$

58 where  $N_d, N_p$  denote the node set output by `mehenDi` at the default parameters and parameter set  $p$  respectively.  
 59  $Path\_length(N_{ai}, N_{bk})$  denote the length of the path between the nodes  $N_{ai}$  and  $N_{bk}$ . We are computing the aver-  
 60 age distance per node between the two sets. If the same node is present in both sets, the distance between them would  
 61 be 0. Similarly, if for a node belonging to one set, there exists a node in the other set which is an ancestor/descendant for  
 62 it, then the distance is computed by calculating the length of the path between them on the tree. On the other hand, if  
 63 there is no ancestor/descendant for a node in the other set, then the distance is the length of the path from the root to  
 64 that node with 1 added. 1 is added since this would be the lowest height node in the other set. The nodes that don't have  
 65 an ancestor or descendant in the other set, can be the largest contributing factor to the distance metric and can create  
 66 asymmetry for the overall distance metric aka  $dist(N_a, N_b) \neq dist(N_b, N_a)$ , as they do not directly have a counterpart  
 67 in the other set. This can skew the metric, depending on the set w.r.t which distance is computed, especially if that set  
 68 consists of nodes that represent unique branches in the tree. To balance this, our final distance metric  $\mathcal{D}(N_d, N_p)$  is the  
 69 average of  $dist(N_d, N_p), dist(N_p, N_d)$ .

70 **S2 Note**

71 **S2.1 Comparing biological and inferential relative variance and its incorporation by Swish**

72 There exists biological variance for a gene across samples in an experiment, with overdispersion modelled using a negative  
 73 binomial/quasi-Poisson distribution in many parametric differential testing methods[2], [1]. However, the inferential  
 74 replicates used by TreeTerminus and `mehenDi` are generated for each biological sample. These inferential replicates provide  
 75 a measure of how certain we are about the abundance estimates for a biological sample using the metric of inferential

76 relative variance (infRV). The inferential replicates mirror the technical replicates, for which the Poisson distribution is  
77 assumed (mean same as variance); any deviation from this trend is captured by infRV, with a higher value of this metric  
78 denoting a high uncertainty associated with the inference of that transcript's abundance. The infRV is computed for  
79 each node (which can be a gene, a transcript, or a transcript-group) for a given biological sample using the inferential  
80 replicates. This metric enables our method to create tree(s) for each biological sample individually, such that uncertainty  
81 decreases upon ascending the tree(s) or across all biological samples in an RNA-seq experiment. Note that inferential  
82 variance for a sample can be high or low regardless of the biological variability seen in the experiment. Swish tries to takes  
83 into account both types of variance in its inference procedure: the biological variance is involved when computing the  
84 Wilcoxon statistic (for the two group mode) while inferential variance per sample is taken into account when integrating  
85 the base test statistic over inferential replicates.

## 86 **S2.2 Role of equivalence classes in capturing inferential uncertainty**

87 The equivalence class is data-dependent and represents a relationship between the set of reads and transcripts, where each  
88 equivalence class contains the reads that map to the same set of transcripts. Thus, equivalence classes are a function of the  
89 data and mapping/alignment algorithm, and we do not have a direct choice in the presented method over the equivalence  
90 relation that is defined and the classes that are produced. That is, the equivalence classes are a deterministic function of  
91 the input data and the alignment algorithm applied. The equivalence classes encapsulate all read-to-transcript mapping  
92 uncertainty, conditional on the mapping/alignment method.

93 Ideally, if each read is mapped uniquely to only a single transcript, then there will be no uncertainty, and the number  
94 of equivalence classes will be equal to the total number of unique transcripts that are mapped by the entire read set. The  
95 only way an equivalence class may be wrongly formed is if the reads are mapped incorrectly, which is a function of the  
96 mapping algorithm. In this case, any quantification algorithm and the corresponding downstream analysis will suffer.

97 It is possible that inferential replicates might not be able to fully capture the uncertainty profile for certain transcripts  
98 due to the limitations of a particular posterior/bootstrap sampling algorithm, but they still provide much more information  
99 than a point estimate.

## 100 **S2.3 Effect of batch effects in capturing uncertainty and on mehenDi output**

101 There can only be two potential places where batch effects might pose an issue, namely, tree construction and downstream  
102 differential testing. The batch effects can alter the counts. When it comes to tree construction, the trees are constructed  
103 using the metric inferential relative variance, which is computed for each sample separately, thus minimizing the issues  
104 which occur due to batch effects, which occur when the counts across samples are compared. Note that the InfRV by  
105 construction is stabilized across the mean count. While subtle differences in uncertainty structure might be observed for  
106 a given sample due to batch effects, we do not anticipate nor have we ever observed substantial changes in the final tree

107 structures that are obtained across samples. For the differential analysis, we don't think batch effects would pose an issue  
108 that would be unique to our analysis. We can use mehenDi in combination with a testing procedure that controls for  
109 batch using standard methods (Swish can also be run stratified across batch, or inferred batch variation can be regressed  
110 out of inferential replicate count matrices). We recommend to control for batch effects in the differential testing method if  
111 it present in the diagnostic plots (e.g. PCA). Moreover, we provide an explicit example of such in the current manuscript.  
112 Specifically, in this manuscript, we have analyzed the ChimpBrain dataset which shows evidence for batch effects. We use  
113 the p-values computed by Swish accounting for batch-effects.

114 **References**

115 [1] Mark D Robinson, Davis J McCarthy, Gordon K Smyth. edgeR: a Bioconductor package for differential expression  
116 analysis of digital gene expression data. *Bioinformatics* 26(1):139–140, 2010.

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118 data with DESeq2. *Genome biology* 15:1–21, 2014.

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120 inferential replicate counts. *Nucleic Acids Research* 47(18):e105–e105, 2019.

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124 inferential replicate counts. *Bioinformatics* 33(14):i142–i151, 2016.

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126 counts. *iScience* 26(6), 2023.

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128 quantification of transcript expression. *Nature methods* 14(4):417–419, 2017.

129 **S3 Figures**

Figure S1: Distribution of the p-values for the leaf and inner nodes on the null simulations, when the hypothesis testing is carried out separately.

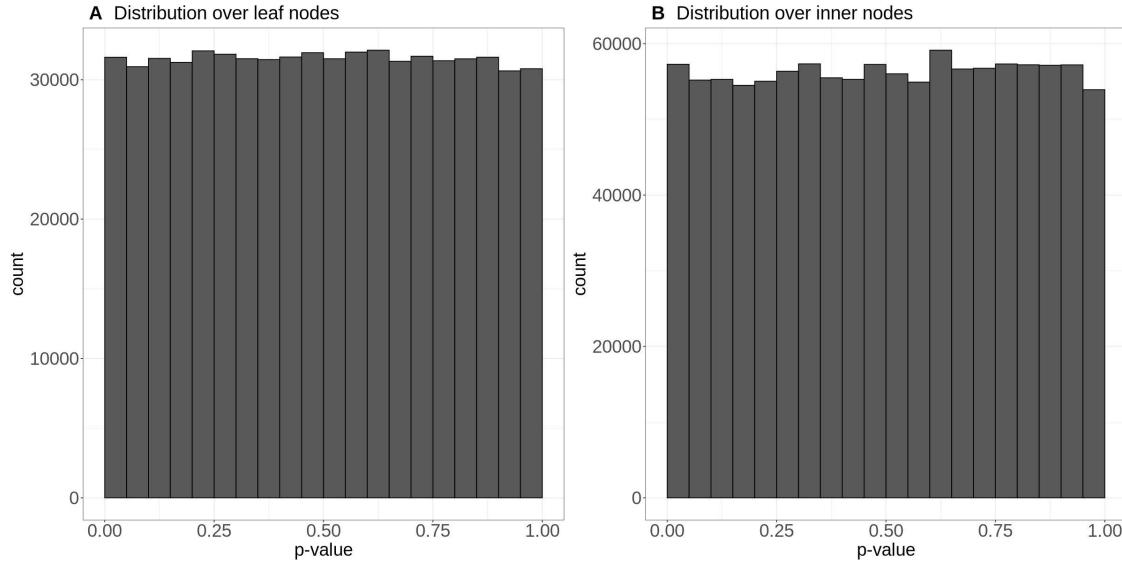


Figure S2: UpSet plot covering the number of true positive transcripts that are covered by the nodes that are output by the different methods for the `BrSimNorm` dataset nominal FDR.

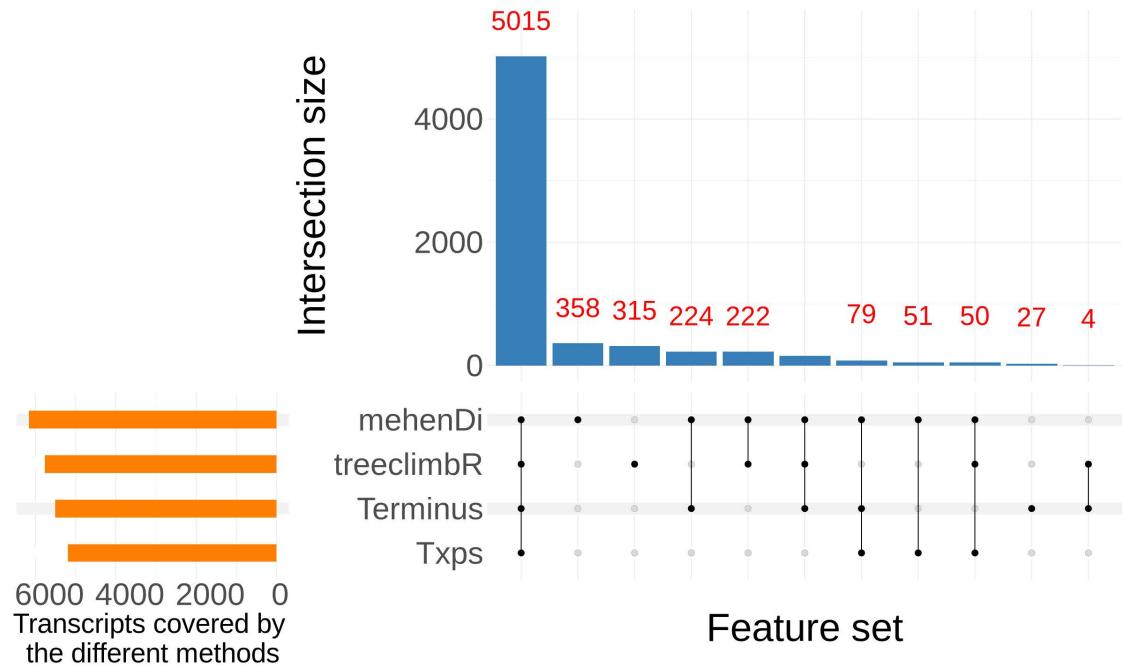


Figure S3: UpSet plot covering the number of true positive transcripts that are covered by the nodes that are output by the different methods for the `BrSimLow` dataset at the 0.01 nominal FDR.

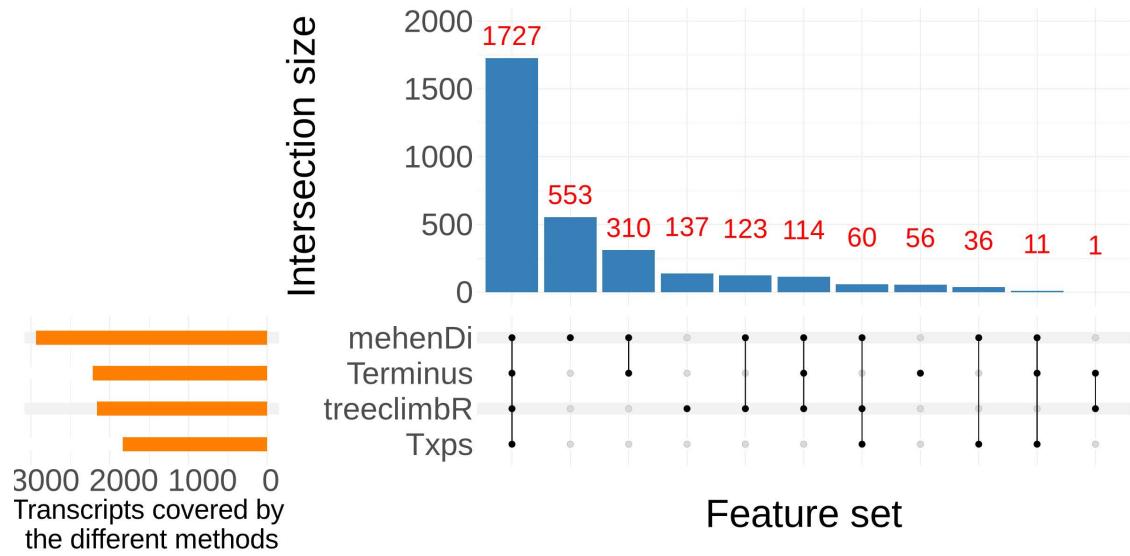


Figure S4: True Positive Rates and Empirical False Discovery Rates at the different nominal FDR thresholds by individually varying the parameters `minP` and `mIRvThresh` for the `BrSimNorm` dataset. Both the metrics have been rounded to 3 decimal places

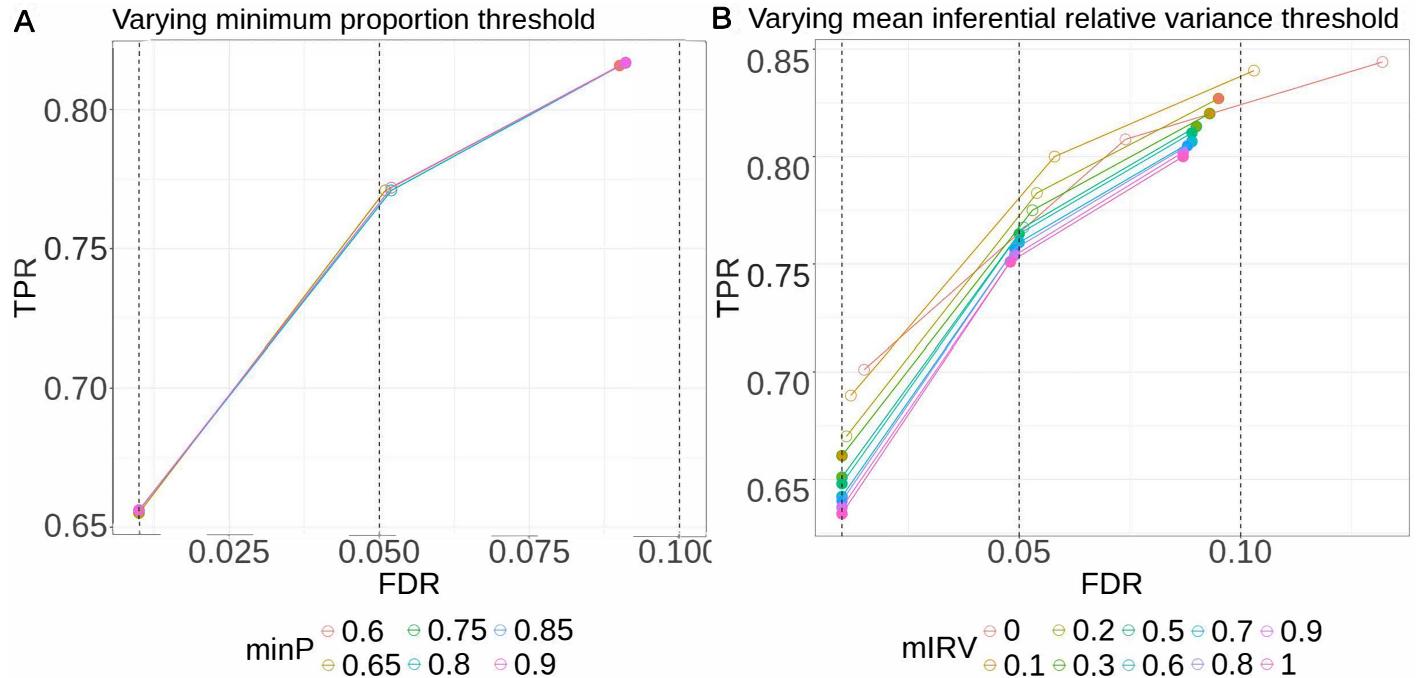


Figure S5: Example of a `mehenDi` node that is overaggregated in `Terminus` for the `BrSimNorm` dataset. (A) Subtree representing the transcripts covered by the `Terminus` group. (B) Inferential replicates for the `Terminus` group. (C) Inferential replicates for the selected node output by `mehenDi`.

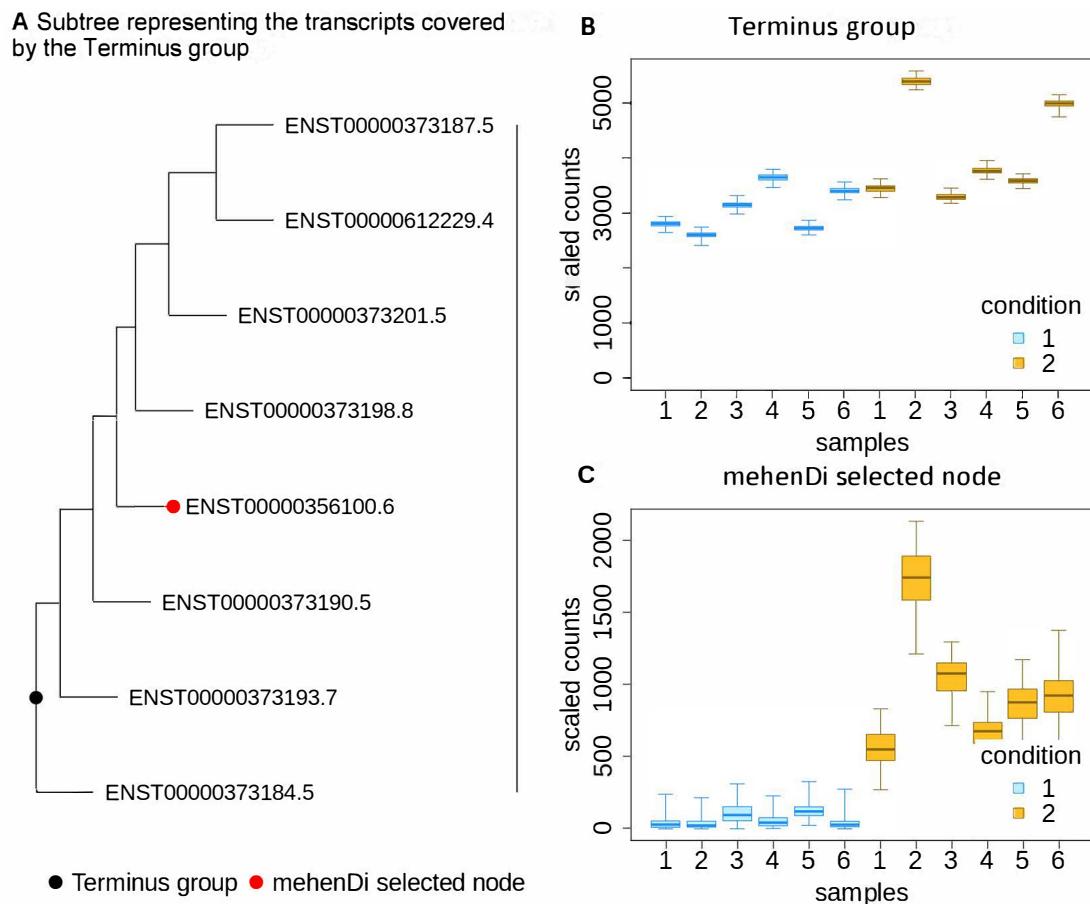


Figure S6: Example of a `mehenDi` node that is overaggregated in `Terminus` for the `BrSimNorm` dataset. (A) Subtree representing the transcripts covered by the `Terminus` group. (B) Inferential replicates for the `Terminus` group. (C) Inferential replicates for the selected node output by `mehenDi`.

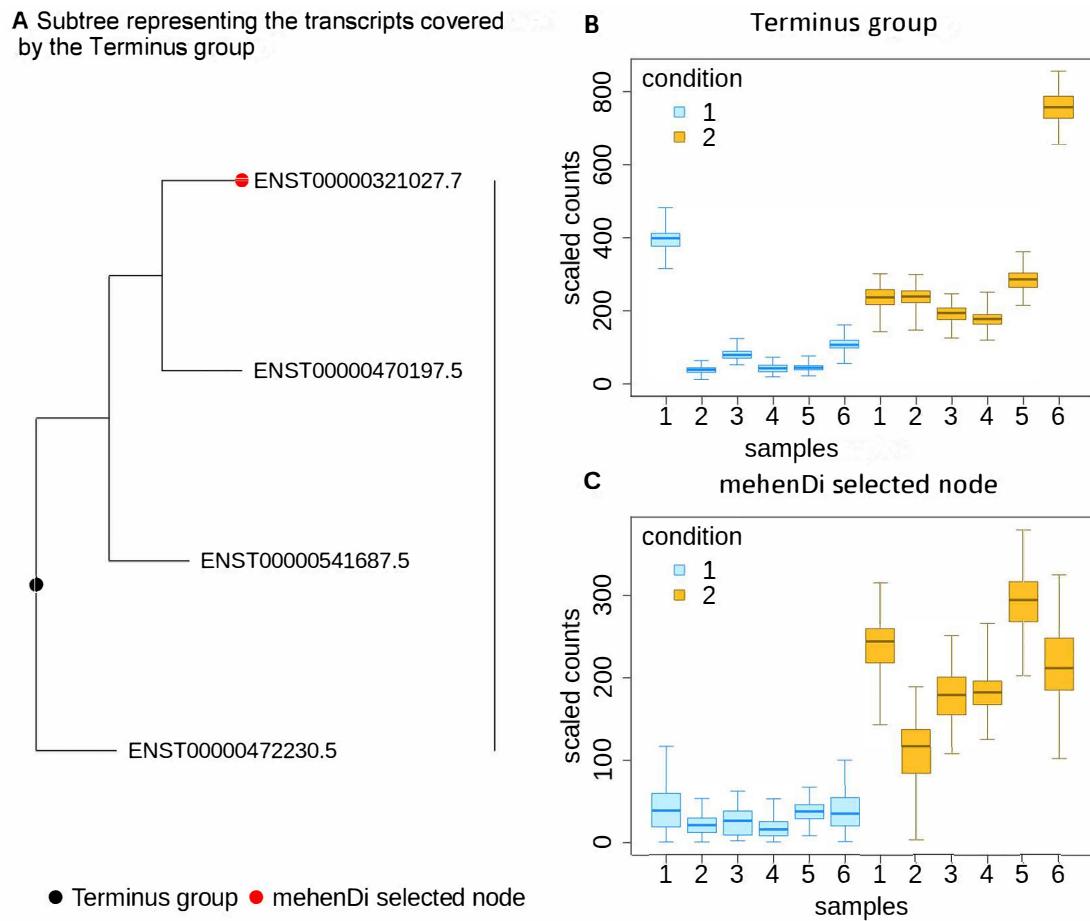


Figure S7: Example of a `mehenDi` node that is not aggregated enough in `Terminus` for the `BrSimNorm` dataset. (A) Subtree representing the transcripts covered by the `mehenDi` group. (B) Inferential replicates for the `Terminus` group. (C) Inferential replicates for the selected node output by `mehenDi`.

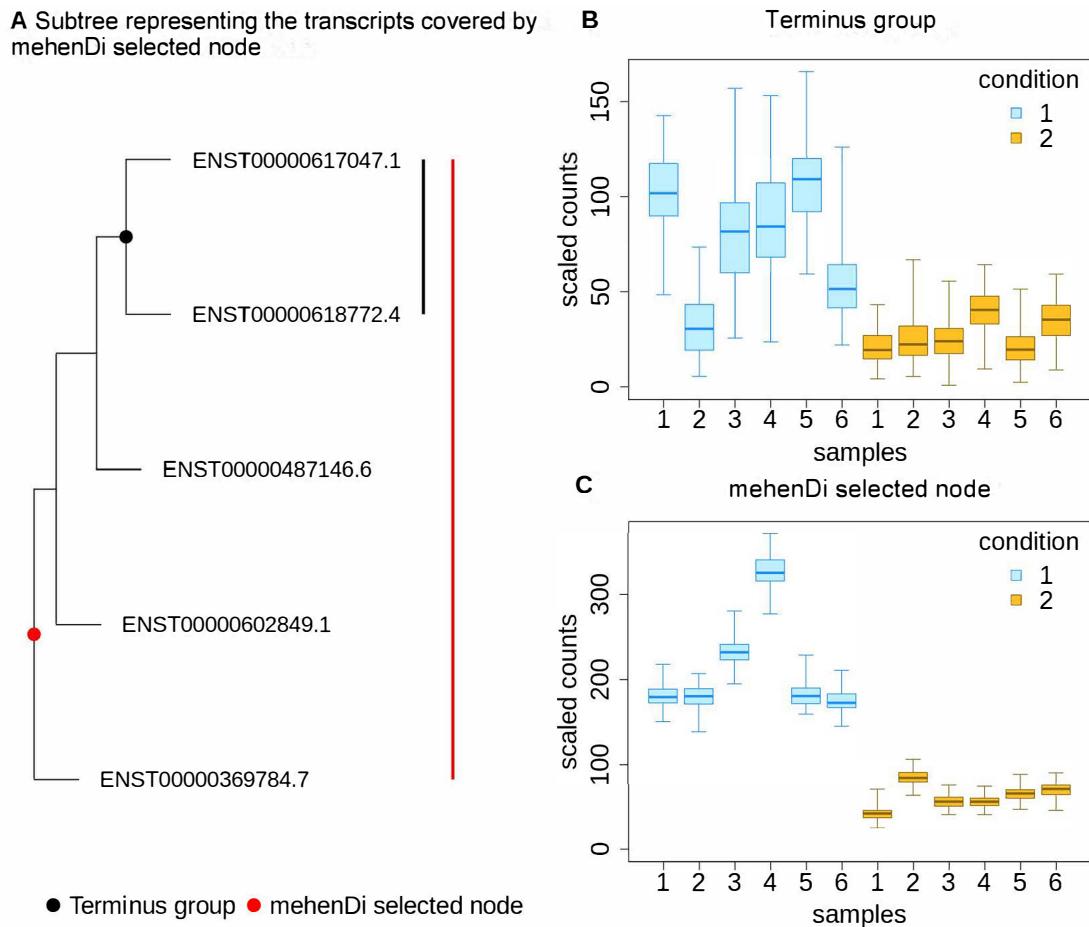


Figure S8: Example of a `mehenDi` node that is not aggregated enough in `Terminus` for the `BrSimNorm` dataset. (A) Subtree representing the transcripts covered by the `mehenDi` group. (B) Inferential replicates for the `Terminus` group. (C) Inferential replicates for the selected node output by `mehenDi`.

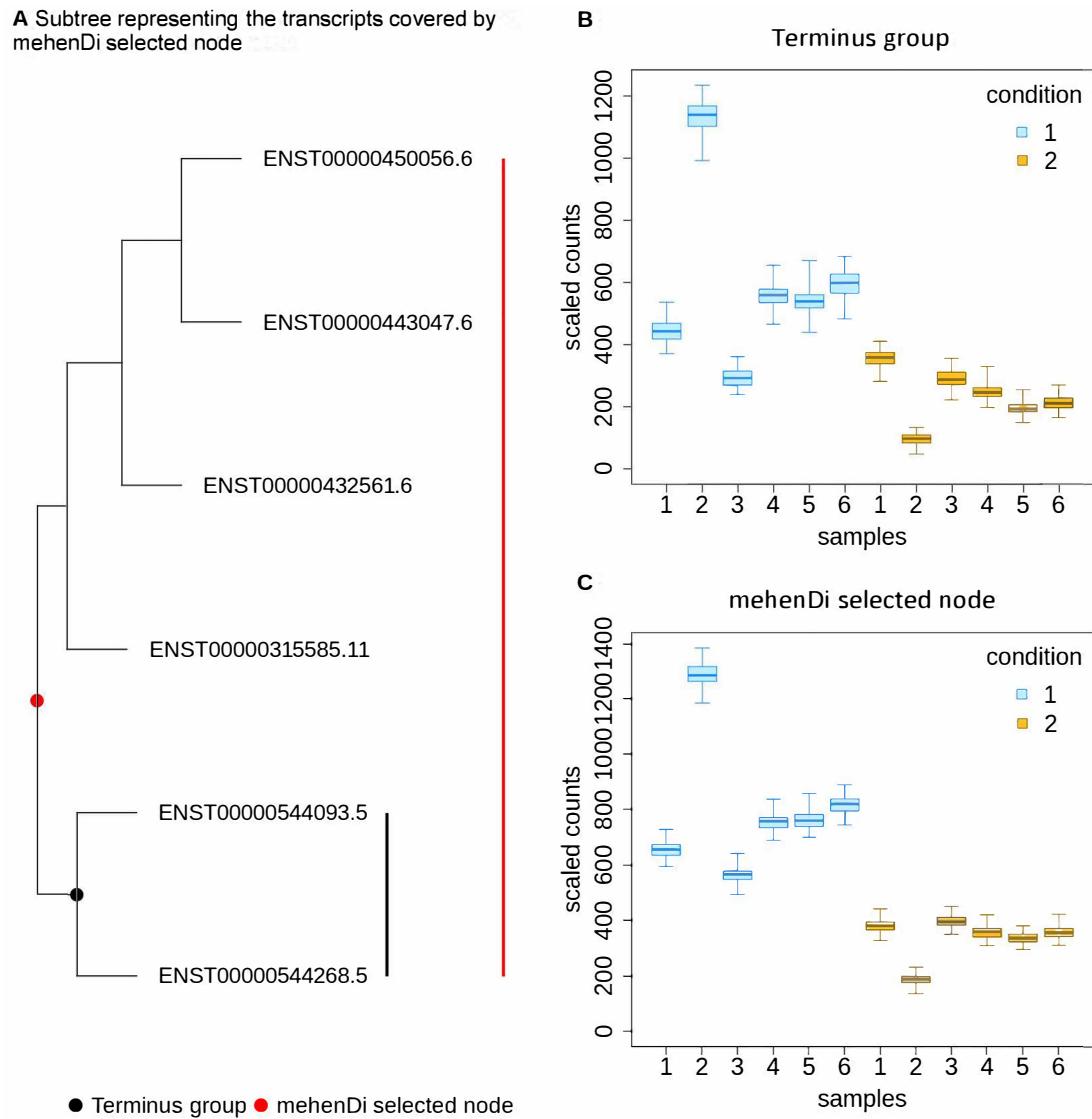


Figure S9: Examination of error metrics for the unique nodes obtained for `treeclimbR` when doing `treeclimbR` vs `Txps` analysis at the different nominal FDR thresholds. We vary the magnitude of log fold change (LFC) and plot the empirical FDR and the total number of nodes that are left after filtering the unique nodes based on LFC.

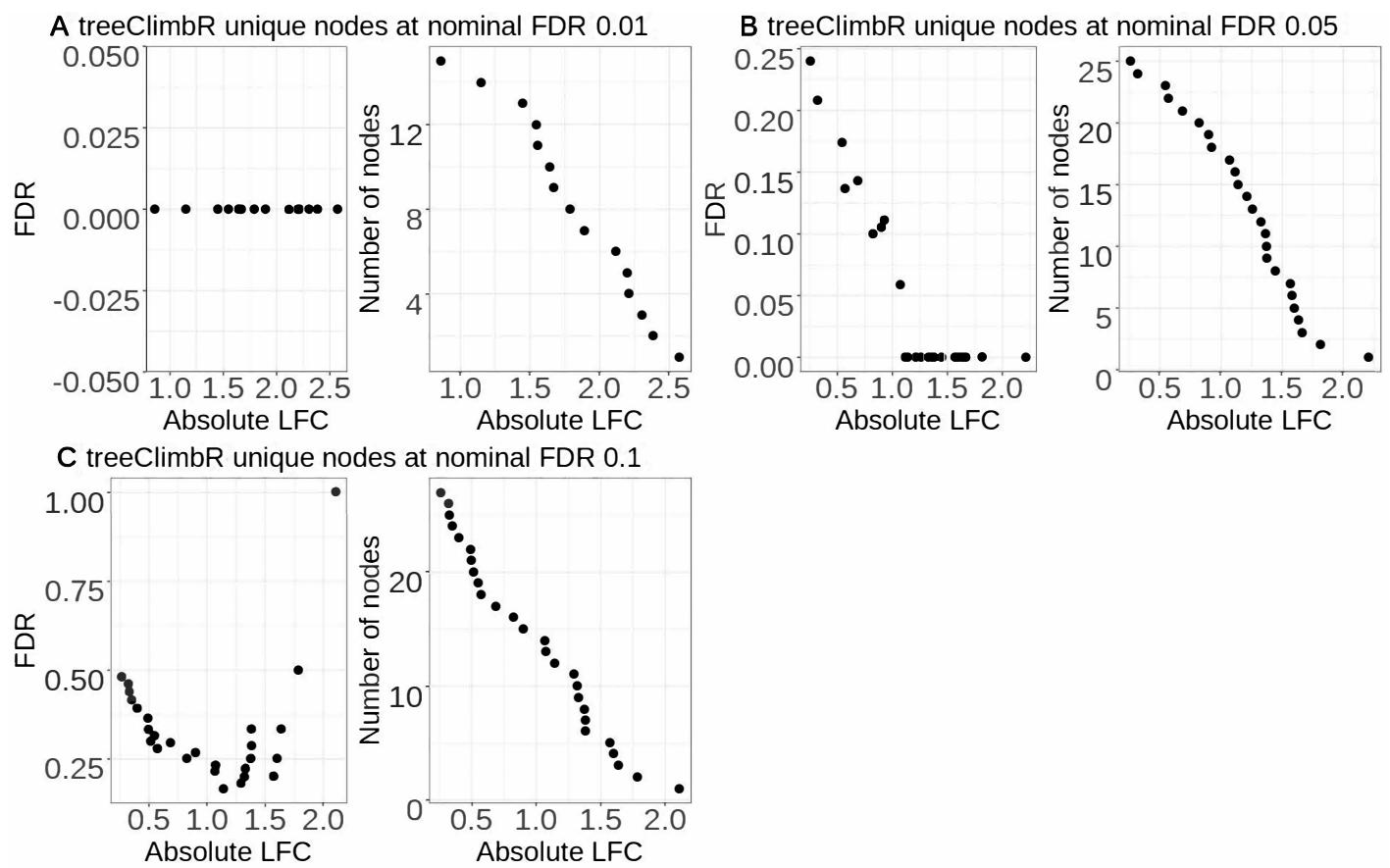


Figure S10: Examination of error metrics for the unique nodes obtained for `Txps` when doing `treeclimbR` vs `Txps` analysis at the different nominal FDR thresholds. We vary the magnitude of log fold change (LFC) and plot the empirical FDR and the total number of nodes that are left after filtering the unique nodes based on LFC.

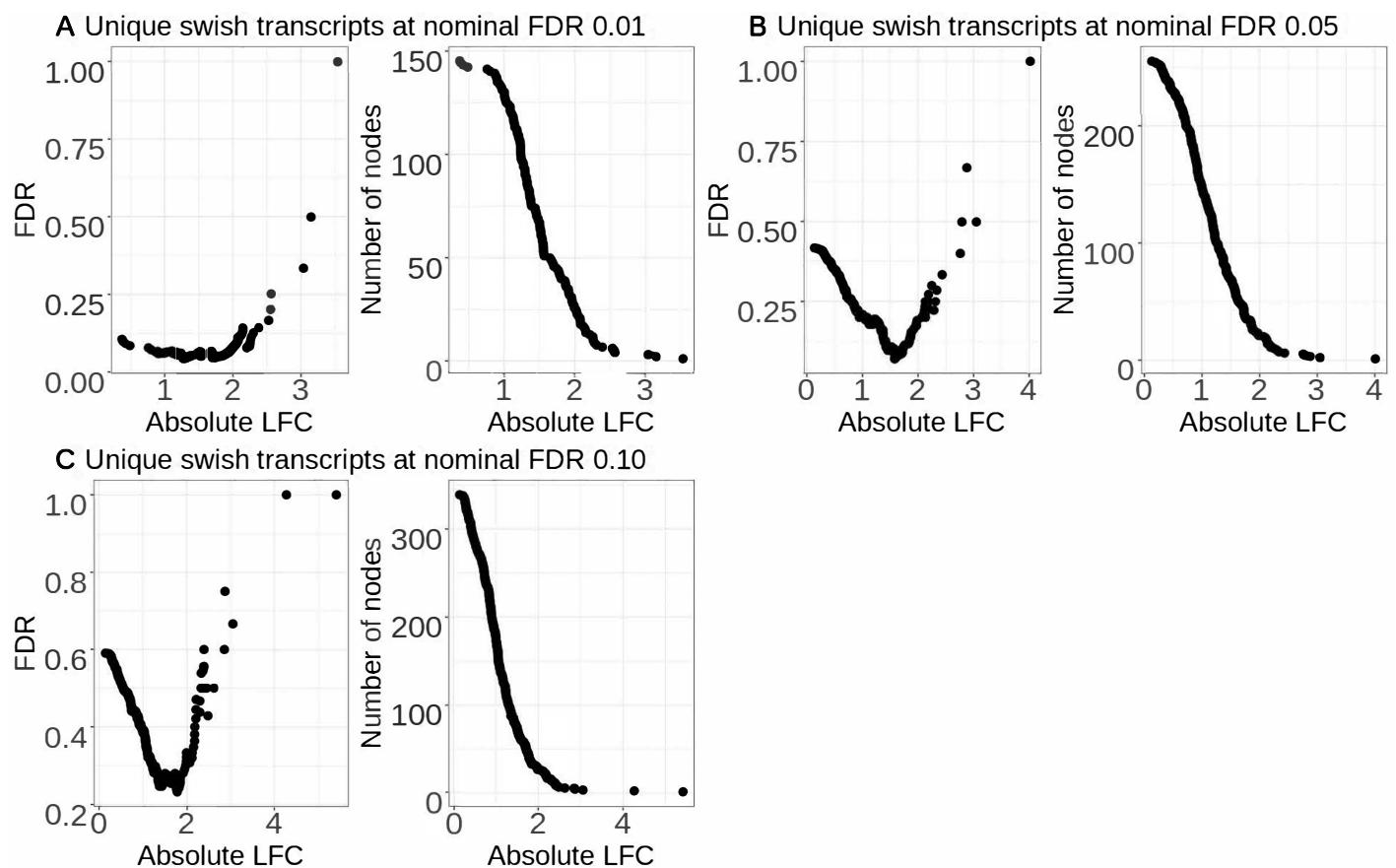


Figure S11: The first two dimensions of the PCA using the top 1000 variable features for the `MouseMuscle` dataset

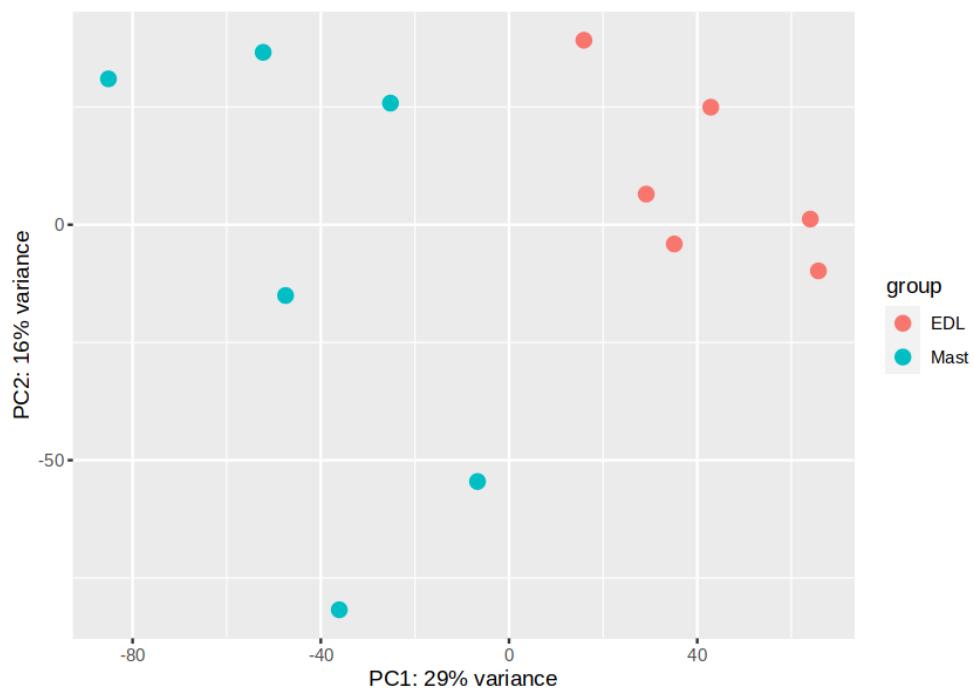


Figure S12: The average distance between the nodes obtained for `mehenDi` using default parameters and varying `minP` and `mlrvThresh` individually for the `MouseMuscle` dataset.

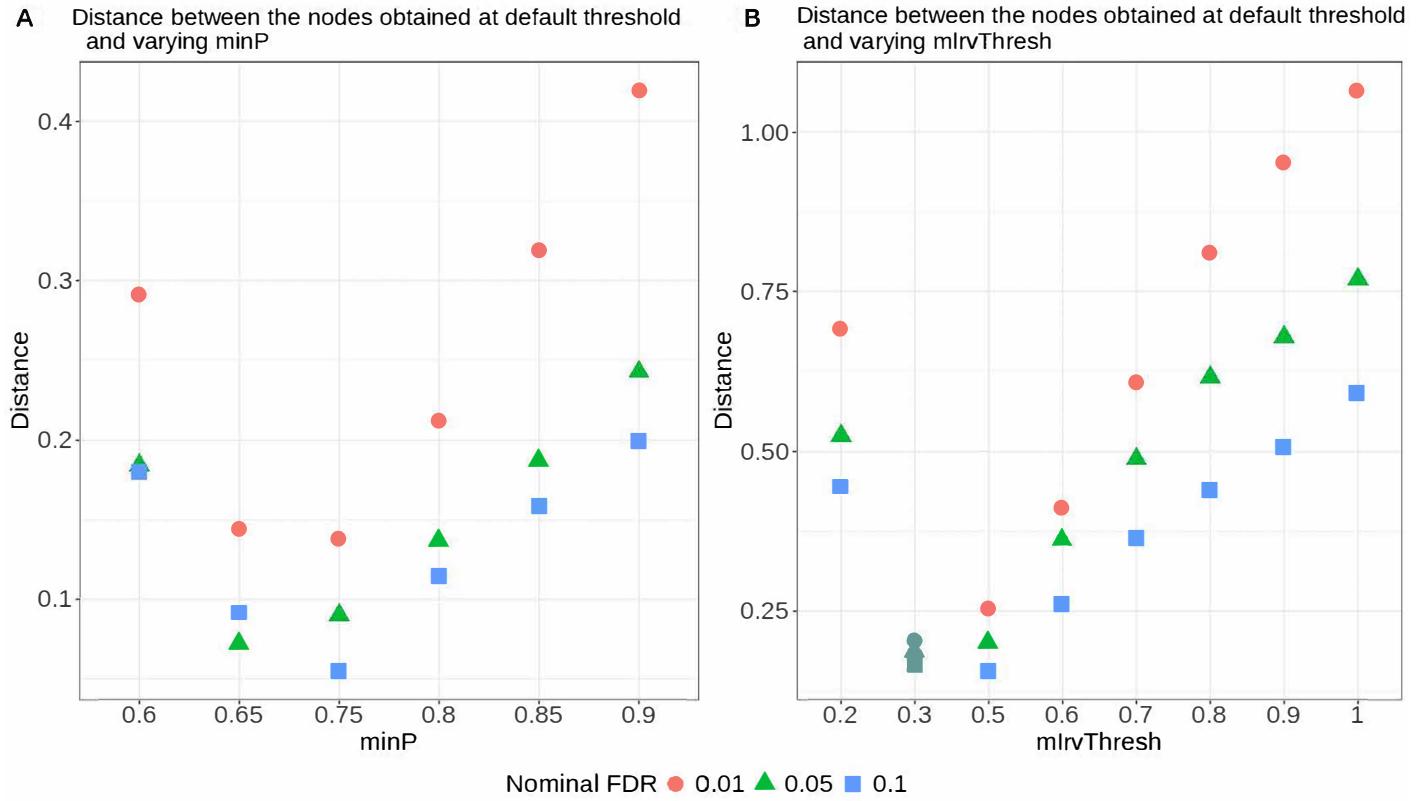


Figure S13: UpSet plot covering the number of transcripts that are covered by the nodes that are output by the different methods for the `MouseMuscle` dataset.

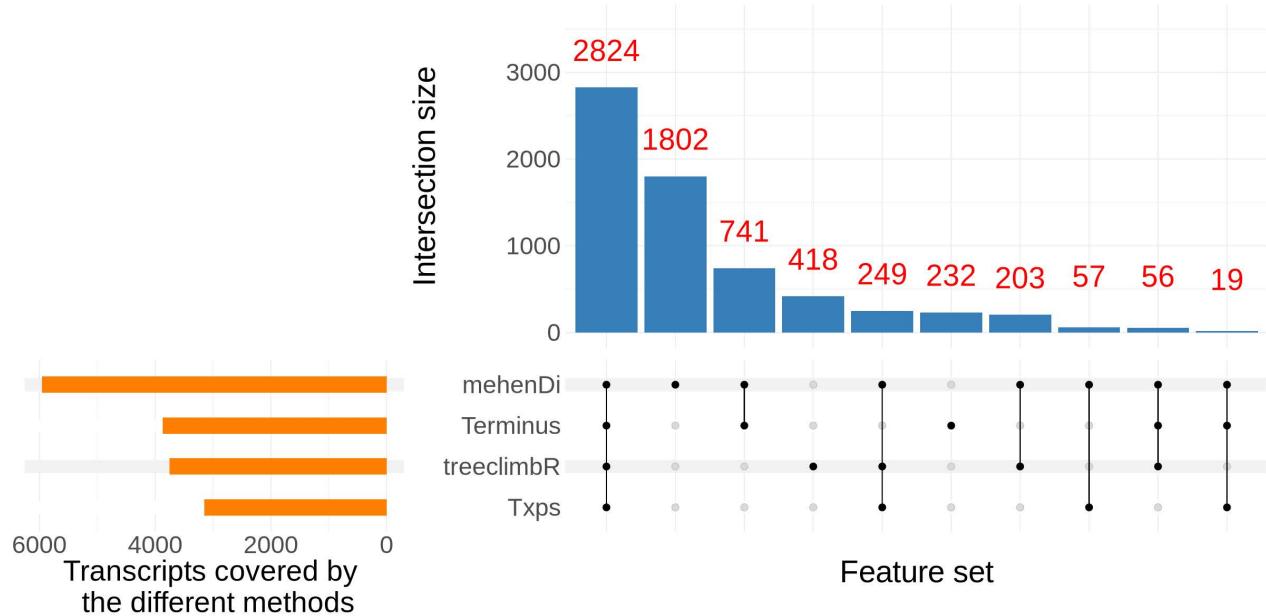


Figure S14: Examining the transcript profile for the gene *Hmcn2* in the **MouseMuscle** dataset. A) Transcripts in a pileup style. B) Tree representing the transcripts covered by the gene *Hmcn2*, with the red node representing the transcripts covered by the **mehenDi** selected node. C) Inferential replicates for the transcript ENSMUST00000138821.7, which had the lowest p-value among all the transcripts in the tree. D) Inferential replicates for the **mehenDi** selected node.

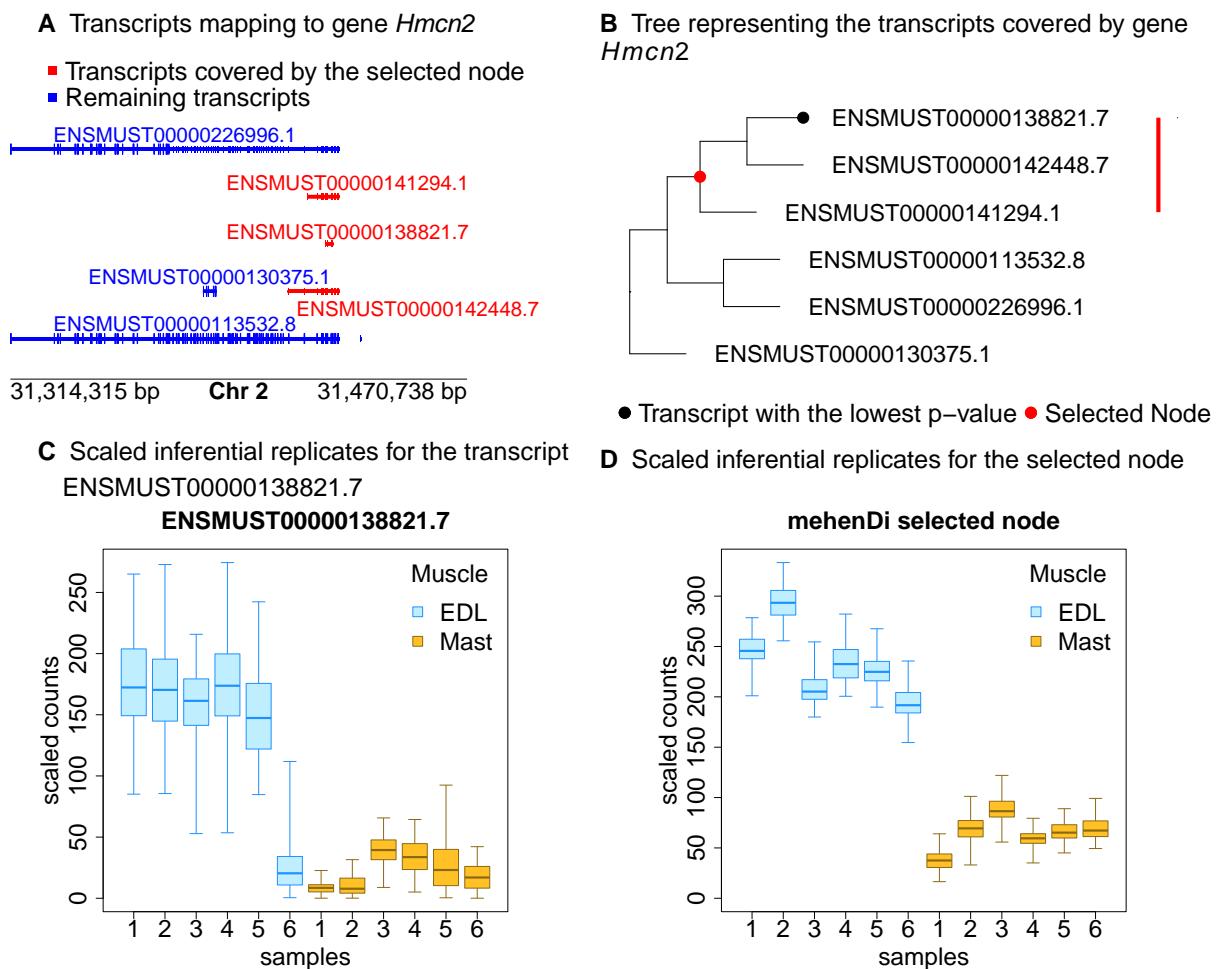


Figure S15: Examining the transcript profile for the gene *Emid1* in the MouseMuscle dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *Emid1*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000151906.7, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

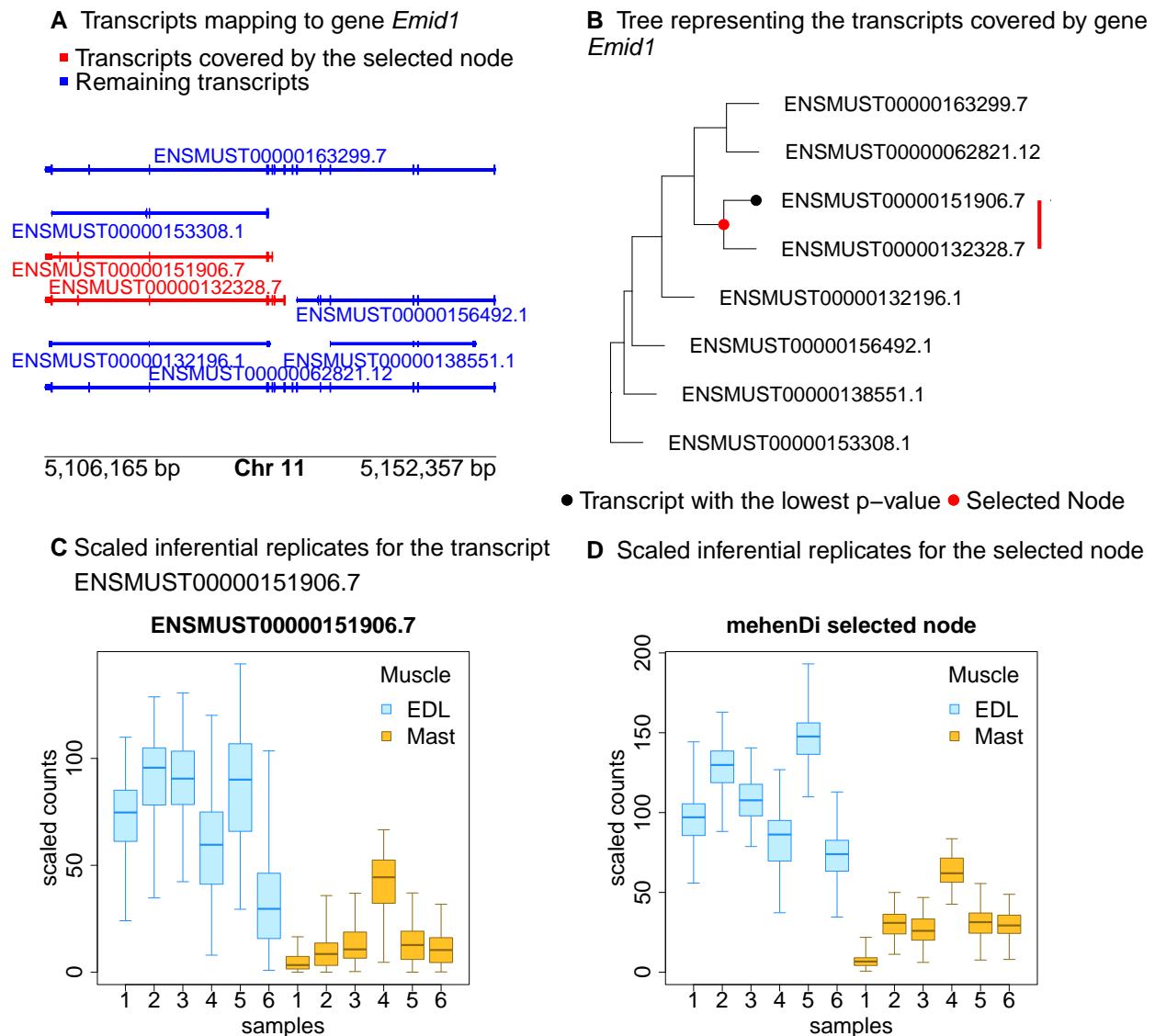
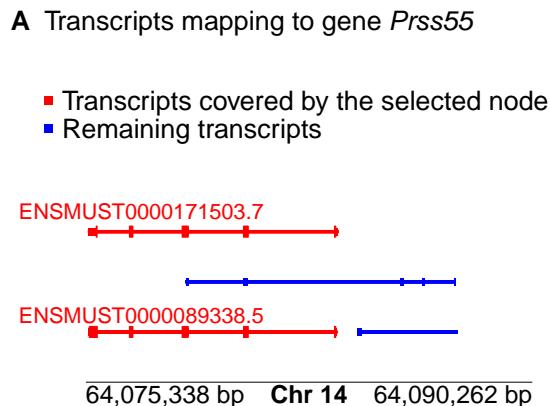
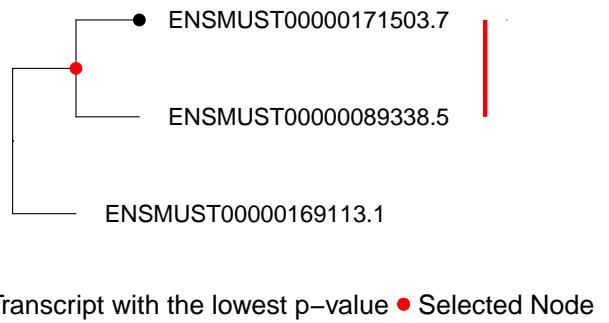


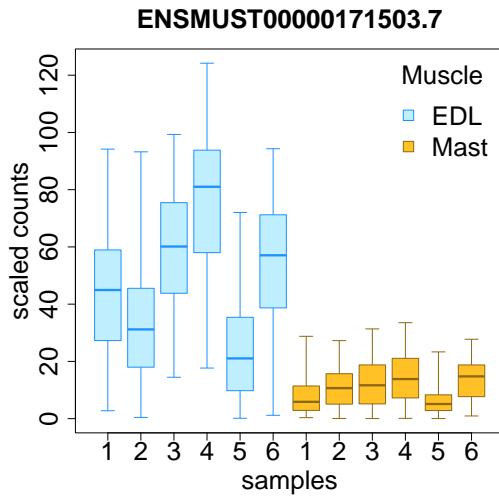
Figure S16: Examining the transcript profile for the gene *Prss55* in the **MouseMuscle** dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *Prss55*, with the red node representing the transcripts covered by the **mehenDi** selected node. (C) Inferential replicates for the transcript ENSMUST00000171503.7, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the **mehenDi** selected node.



**B Tree representing the transcripts covered by gene *Prss55***



**C Scaled inferential replicates for the transcript ENSMUST00000171503.7**



**D Scaled inferential replicates for the selected node**

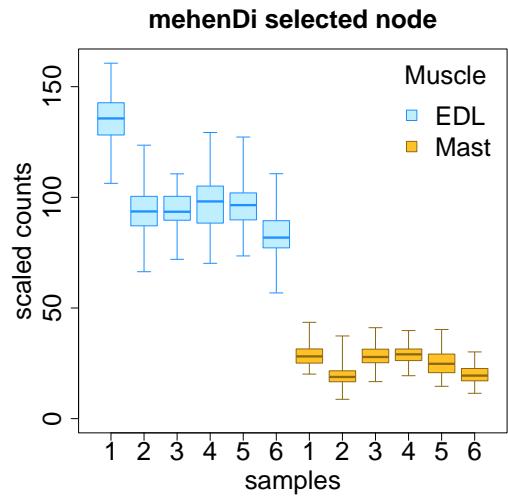
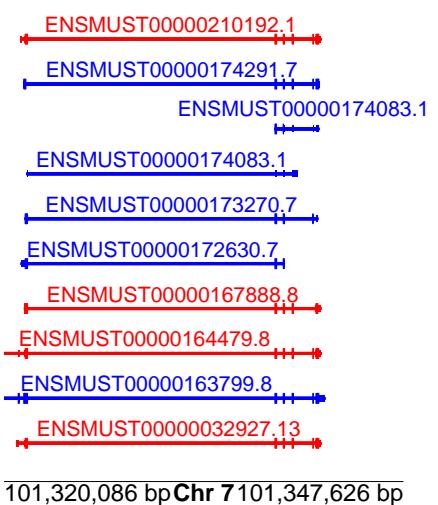


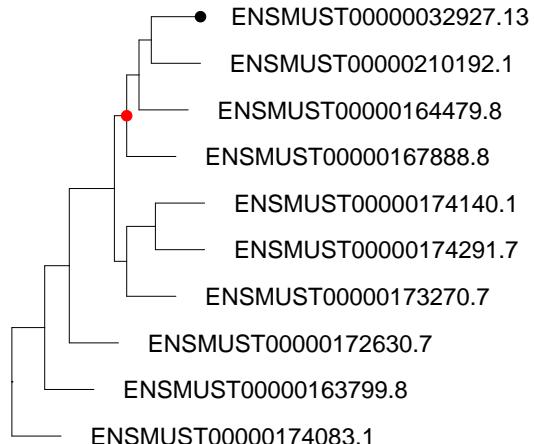
Figure S17: Examining the transcript profile for the gene *Stard10* in the MouseMuscle dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *Stard10*, with the red node representing the transcripts covered by the mehenDi selected node. (C) Inferential replicates for the transcript ENSMUST00000032927.13, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the mehenDi selected node.

**A** Transcripts mapping to gene *Stard10*

- Transcripts covered by the selected node
- Remaining transcripts

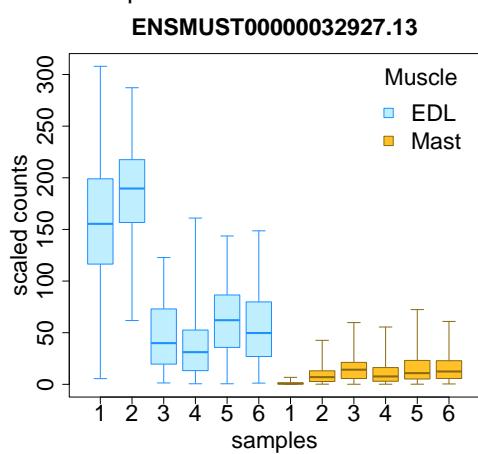


**B** Tree representing the transcripts covered by gene *Stard10*



● Transcript with the lowest p-value ● Selected Node

**C** Scaled inferential replicates for the transcript ENSMUST00000032927.13



**D** Scaled inferential replicates for the selected node

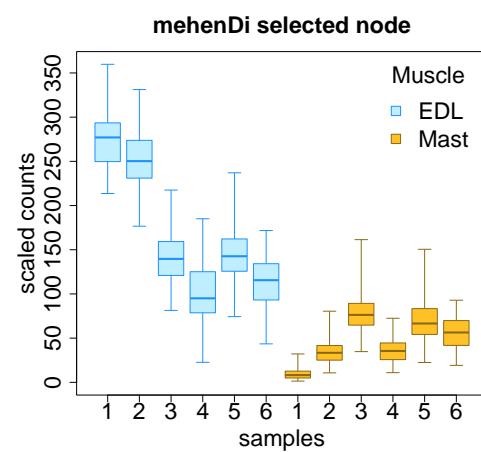


Figure S18: Examining the transcript profile for the gene *Syk* in the MouseMuscle dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *SYK*, with the red node representing the transcripts covered by the *mehenDi* selected node. (C) Inferential replicates for the transcript ENSMUST00000055087.6, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the *mehenDi* selected node.

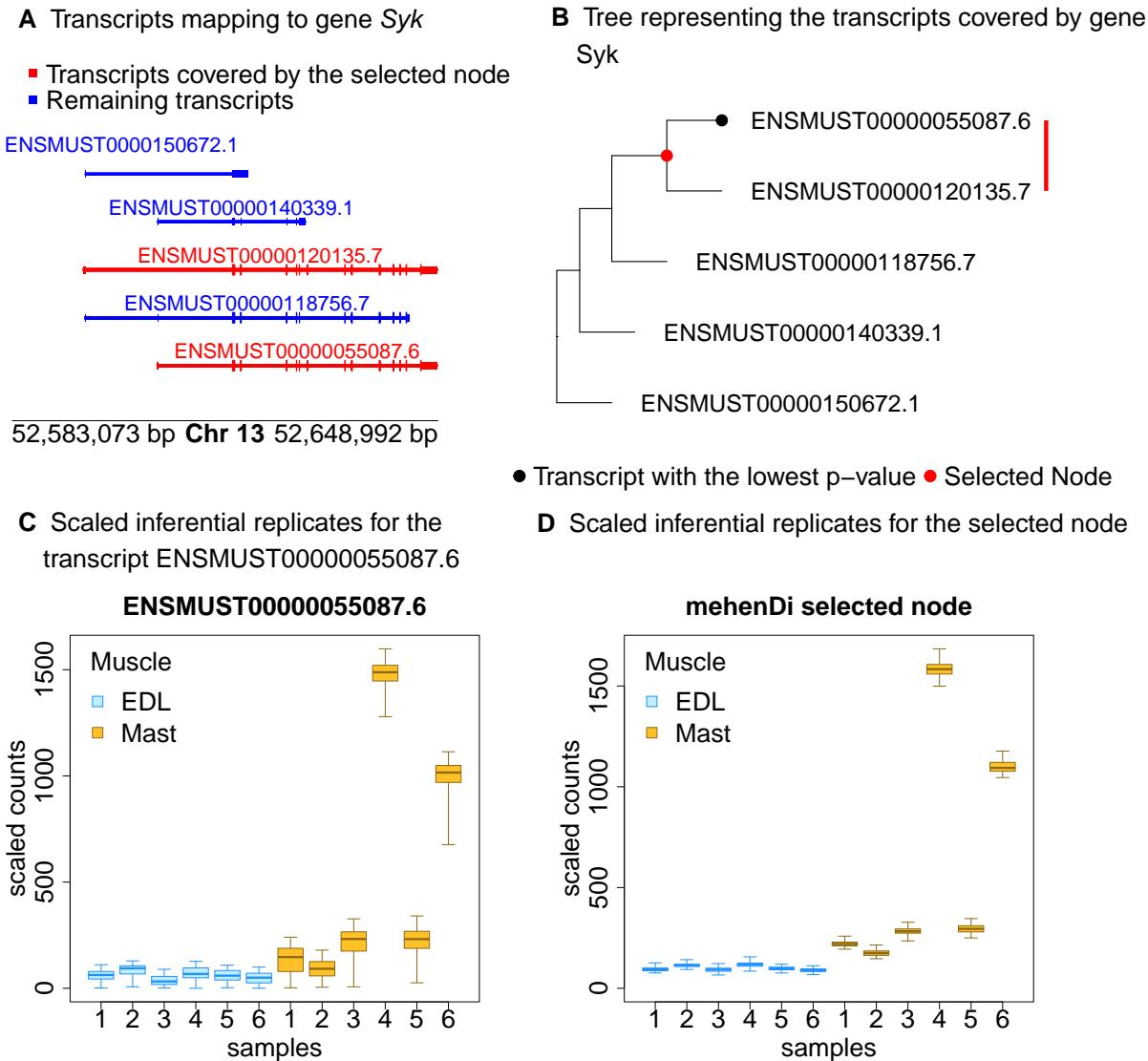


Figure S19: UpSet plot covering the number of transcripts that are covered by the nodes that are output by the different methods for the ChimpBrain dataset.

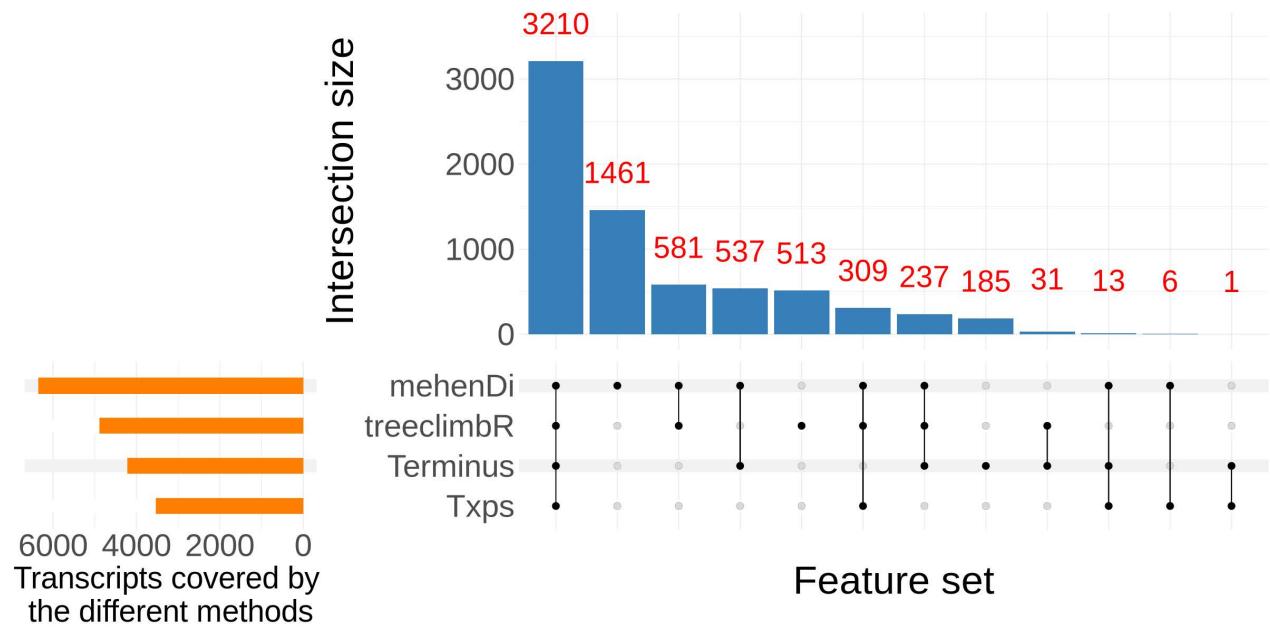


Figure S20: Examining the transcript profile for the gene *CABIN1* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *CABIN1*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000103768, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

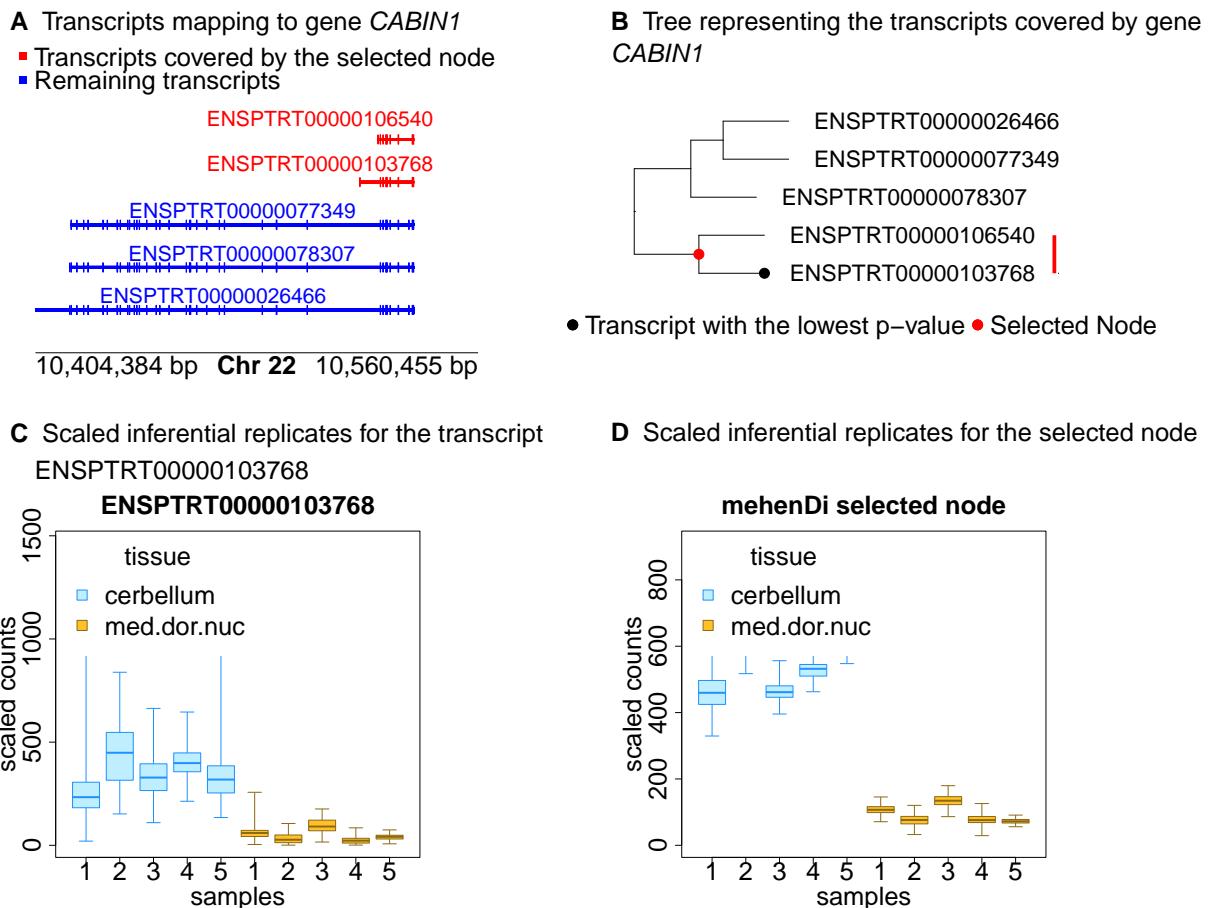


Figure S21: Examining the transcript profile for the gene *CATSPERG* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *CATSPERG*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000020240, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

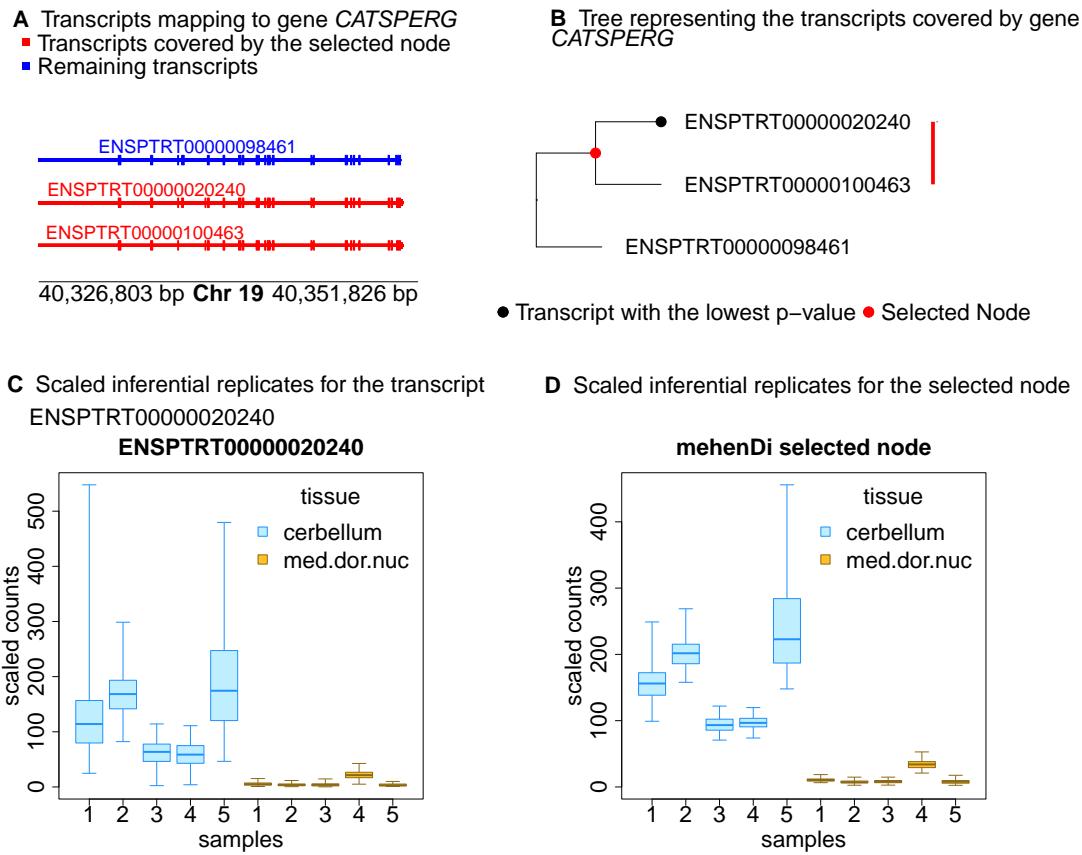


Figure S22: Examining the transcript profile for the gene *EYA1* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *EYA1*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000097751, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

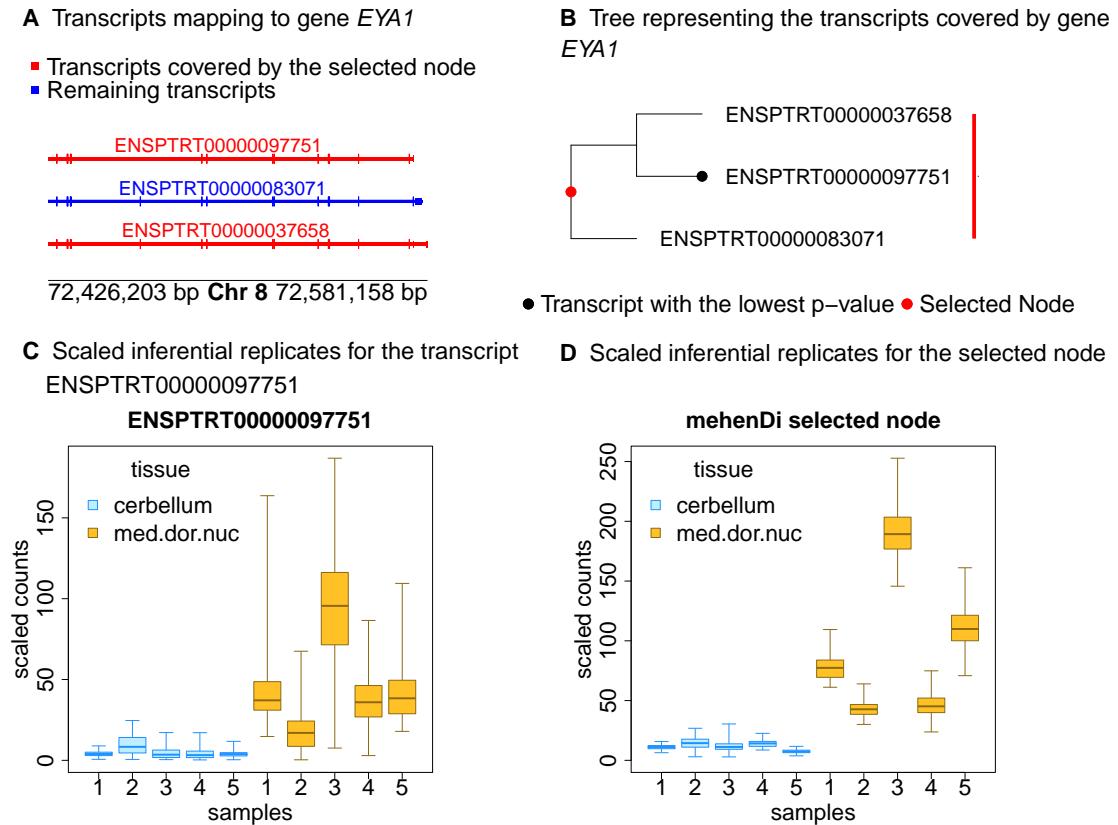


Figure S23: Examining the transcript profile for the gene *MCF2* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *MCF2*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000105901, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

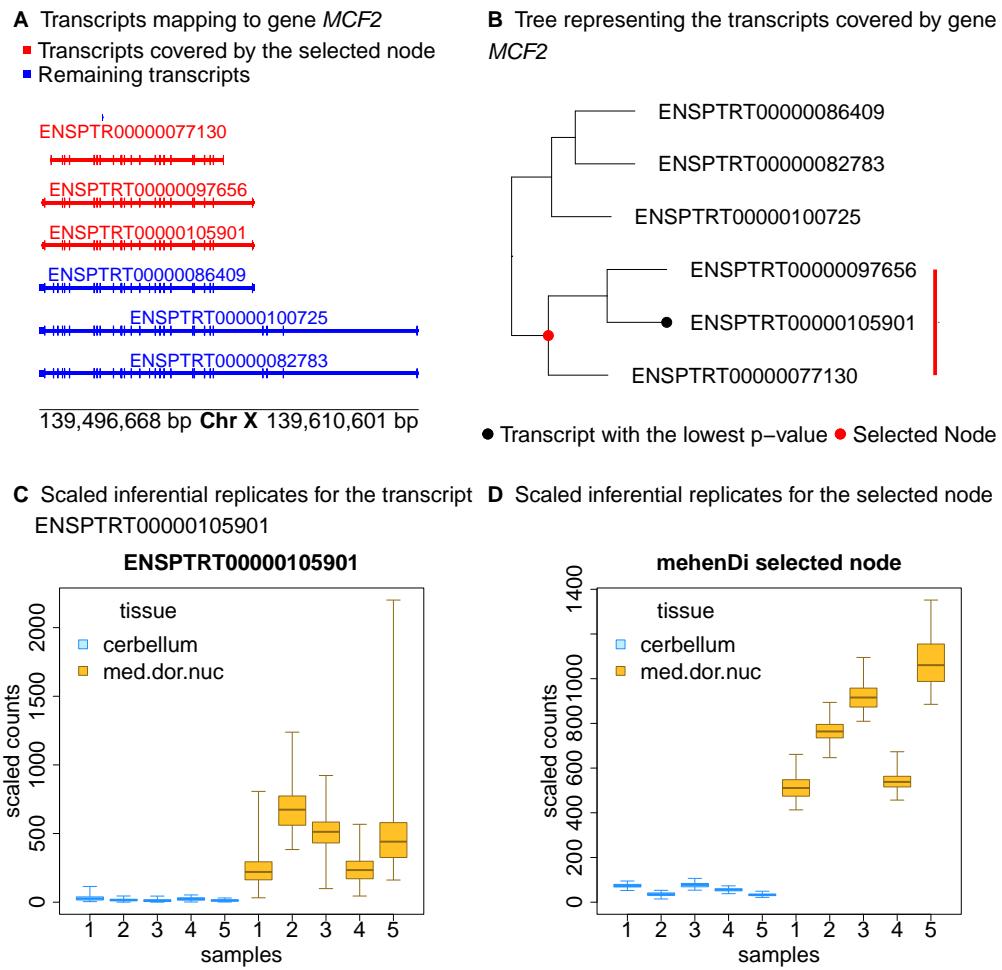


Figure S24: Examining the transcript profile for the gene *MYO5C* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *MYO5C*, with the red node representing the transcripts covered by the `mehenDi` selected node. (C) Inferential replicates for the transcript ENSMUST00000013074, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the `mehenDi` selected node.

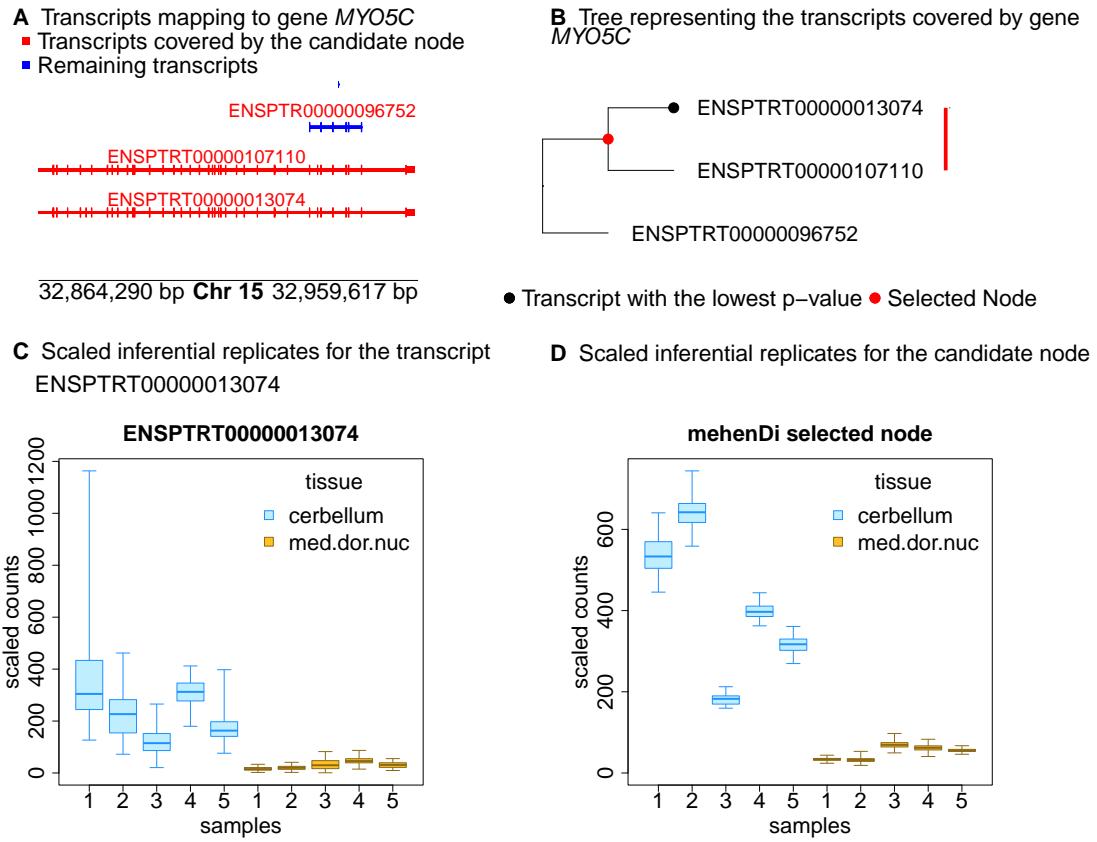


Figure S25: Examining the transcript profile for the gene *PIEZ02* in the ChimpBrain dataset. (A) Transcripts in a pileup style. (B) Tree representing the transcripts covered by the gene *PIEZ02*, with the red node representing the transcripts covered by the *mehenDi* selected node. (C) Inferential replicates for the transcript ENSMUST00000094046, which had the lowest p-value among all the transcripts in the tree. (D) Inferential replicates for the *mehenDi* selected node.

