Prediction of individual genetic risk to disease from genome-wide association studies

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Empirical studies suggest that the effect sizes of individual causal risk alleles underlying complex genetic diseases are small, with most genotype relative risks in the range of 1.1–2.0. Although the increased risk of disease for a carrier is small for any single locus, knowledge of multiple-risk alleles throughout the genome could allow the identification of individuals that are at high risk. In this study, we investigate the number and effect size of risk loci that underlie complex disease constrained by the disease parameters of prevalence and heritability. Then we quantify the value of prediction of genetic risk to disease using a range of realistic combinations of the number, size, and distribution of risk effects that underlie complex diseases. We propose an approach to assess the genetic risk of a disease in healthy individuals, based on dense genome-wide SNP panels. We test this approach using simulation. When the number of loci contributing to the disease is >50, a large case-control study is needed to identify a set of risk loci for use in predicting the disease risk of healthy people not included in the case-control study. For diseases controlled by 1000 loci of mean relative risk of only 1.04, a case-control study with 10,000 cases and controls can lead to selection of ~75 loci that explain >50% of the genetic variance. The 5% of people with the highest predicted risk are three to seven times more likely to suffer the disease than the population average, depending on heritability and disease prevalence. Whether an individual with known genetic risk develops the disease depends on known and unknown environmental factors.

An important benefit from the study of the genetics of human disease is to predict that individuals may have of succumbing to a particular disease. Knowledge of this risk can then be used by the clinician in prevention, diagnosis, prognosis, and treatment. Currently, clinicians use the family history of a patient to help assess their risk of a disease with a known genetic component, with family history formally included in standard international disease classification systems. With modern molecular tools, can we improve on the use of family history to assess genetic risk of disease? For diseases caused by single genes, the answer is obviously “yes,” but for diseases with complex inheritance, the best method to use and the success that might be expected are unclear. The dominant paradigm in human complex-trait genetics has been to map loci affecting disease risk and then to identify the causative mutations. Complex traits are likely to be affected by many genes and mutations, most of which have a small effect on disease risk. The relative risk of disease due to one allele is typically of the order of 1.1 to 2.0 (Ioannidis et al. 2006; Bertram et al. 2007), but observed effect sizes may still represent the upper tail of true effect size. These findings are consistent with the expectation that quantitative complex traits in general are affected by a large number of loci. In species where it is possible to measure the number of loci influencing a trait, around 10–50 loci have been identified, most individually counting for only a few percent, and together accounting for <50% of the genetic variation (Henderson et al. 2004; Jacobsson et al. 2005; Valdar et al. 2006). The number of risk loci underlying complex disease and their effect size must be constrained by the disease parameters of prevalence and heritability, yet, to our knowledge, this relationship has not been explored.

Identification of causal variants and elucidating disease pathways through genetic and functional studies is difficult and time-consuming, particularly if there are many risk loci with small effects. However, knowledge of all risk loci or knowledge of causal variants at any one risk locus is not necessary for the prediction of the risk to disease of individuals in the population. The recent advances in high-density single-nucleotide polymorphism (SNP) technology (Kennedy et al. 2003) have made it possible to conduct genome-wide association studies (Hirschhorn and Daly 2005). These studies have been considered as only a first step to identifying the causal mutations, but the SNPs used in the association study could be useful to create genome-wide predictors of disease or a “genomic profile” (Khoury et al. 2004) for disease risk. Janssens et al. (2006) have investigated predictive testing for complex diseases using multiple genes by simulation. They examined diseases controlled by up to 400 risk loci, but, although they considered a range of risk effects and allele frequencies, they did not consider distributions of risk effects that relate to models of the genetic architecture underlying complex diseases. More importantly, the model that Janssens et al. used implicitly assumed that an individual’s true genetic risk is known without error, so that the correlation between observed genetic risk and disease status is simply the square root of the broad-sense heritability on the observed scale. That is, their study does not deal with the key problem, which is to predict the genetic risk faced by each individual. Pharoah et al. (2002) quantified the proportion of cases in subsets of the population that are at highest genetic risk of disease, assuming a polygenic model for which risk was log-normally distributed, without modeling or estimating the effects of individual risk loci.

The objective of this study is to quantify the accuracy of risk prediction from genome-wide association studies and to quantify...
the true disease risk faced by the people predicted to be most at risk in subsequent samples from the population. To do this we consider models of the underlying genetic architecture assuming realistic distributions of the frequencies and effect sizes of risk loci, constraining the number of risk loci and the mean effect size to be consistent with disease prevalence and heritability. Using these models, we estimate the genetic risk of individuals based on a simulated genome-wide association study (GWAS).

Results

The success of association studies, and also of genomic profiling, depends on the genetic architecture underlying complex diseases. First, we investigate the relationship between the relative risk (RR) of genetic loci and the number of loci that contribute to risk of a disease under constraints of known disease prevalence and heritability. We model the genetic architecture of complex disease by allowing the effect size and frequency of risk allele to vary across loci. We go on to use these results to investigate the possibilities of using multiple risk loci identified in a GWAS to predict risk of disease in a new population cohort.

We consider four disease scenarios based on realistic combinations of disease prevalence, \( K = 0.05 \) or 0.10, and heritabilities of the disease on the observed scale, \( h^2 = 0.1 \) or 0.2. We consider two distributions of frequency of risk alleles underlying the disease (Fig. 1): A uniform distribution of allele frequencies that broadly corresponds to the common-disease common-variant (CDCV) hypothesis in which the frequency of the increasing risk allele was simulated as \( p_i \sim \text{Uniform}(0.01,0.99) \) or a U-shaped distribution, which broadly corresponds the neutral allele hypothesis (Pritchard 2001).

Number of loci underlying complex diseases

For a given number of disease loci we force the effect sizes to be consistent with the disease prevalence and heritability parameters. The average relative risks for fixed numbers of disease loci for the four disease scenarios are given in Figure 2. Summary statistics of the mean and maximum RR, the maximum percentage of genetic variance explained by a single locus, and the percentage of genetic variance explained by extreme frequency risk variants describe the properties and differences of the models (Table 1). For the CDCV model, an average RR of 1.2 corresponds to 40 or more loci. As expected, fewer loci imply larger average RR, and the average risk of loci for the approximate neutral model of evolution is always larger than that for the CDCV model for the same number of loci. Similarly, the maximum percentage of genetic variance explained by a single locus is always larger for the neutral model compared to the CDCV model when the number of risk loci is the same. However, the relationship between the number of disease loci and their average RR is broadly similar for the two models. When 1000 risk loci influence a disease, the maximum contribution to genetic variance of any single locus is only 3%-4%. As expected, as the sibling risk increases, the average RR increases if the number of risk loci is fixed; or, the number of risk loci required increases if the mean RR is held constant.

We derived an analytic expression for number of loci when RR and allele frequency are fixed for all loci (Equation 3), which agrees well with the results of the CDCV model when \( p = 0.5 \) and with the neutral model when allele frequency \( p = 0.1 \); e.g., for \( K = 0.05 \) and \( h^2 = 0.2 \), then for \( p = 0.5 \), the number of loci for fixed relative risks of 1.1, 1.2, 1.4, and 1.6 are 346, 95, 29, and 15, respectively; and for \( p = 0.1 \), the corresponding number of loci are 889, 227, 59, and 28. Different combinations of \( K \) and \( h^2 \) can lead to the same sibling (sib) relative risk (Fig. 2); it is this combined parameter that drives the results. Equation 3 can be used to investigate the impact of \( K \) or \( h^2 \) on the number of loci underlying complex diseases (Fig. 3).

Use of GWAS to predict disease risk

Using our models for the genetic architecture of complex diseases, we go on to investigate prediction of genetic risk to disease from multiple risk loci identified in a GWAS. To do this we simulated a case-control study assuming a single-stage genome-wide association screen with 500,000 SNPs. The number of disease risk loci was fixed at 10, 20, 50, 100, 300, or 1000, and allele frequencies were simulated from either the U-shaped (neutral) or uniform distribution (CDCV). Table 2 summarizes the number of loci selected for prediction of genetic risk and the proportion of variance in log risk that they explain in an independent sample of people.

For all simulated scenarios, when 10,000 cases and controls were used, the accuracy with which the genetic risk of disease was predicted in a new random sample of the population of 1000 individuals was very high (Fig. 4 for CDCV model; results for the neutral model were similar but less conservative). For example, for the CDCV model of a disease with prevalence 0.05 and heritability 0.1 caused by 100 risk loci with average RR of 1.15 (Table 1), the accuracy of prediction was 0.97 (Fig. 4). The prediction equation used 45 loci that explained 94% of the genetic variance (Table 2). As the number of risk loci increases from 100 to 300 to 1000, the accuracy remains above 0.70, even though the average genotype relative risk falls below 1.1 (Fig. 4). The number of loci included in the prediction profile continues to increase as the total number of risk loci increases (Table 1), although the percentage of genetic variation they explain decreases. Even when only 1000 cases and controls were used, the accuracy of prediction was high (>0.7) unless the number of disease loci was >50, corresponding to average RR of disease alleles of <1.2. A GWAS of this size does not have sufficient power to detect risk loci with low average RR, and hence the number of loci selected for inclu-
that even with only 1000 cases and 1000 controls in the association study, most of the true disease loci are selected. When the number of risk loci is high, the mean RR is low, but the distribution of RR means that almost all the genetic variance is explained by a fraction of the risk loci.

Discussion

We have quantified the number of disease loci underlying common disease using realistic parameters and have shown that results from GWAS can be used to identify healthy individuals in the population who are at a substantially increased risk of developing disease, even when individual risk loci confer small relative risks. From our model we first determined the relationship between the number of susceptibility loci underlying a complex disease and their average RR, given the allele frequency distribution of risk alleles, the population prevalence, and the heritability. Our results are robust to the distribution of risk allele frequencies assumed (approximating the CDCV or neutral model). We assume additive gene action on the log risk scale (multiplicative gene action on the risk scale), that loci act independently and that there is no linkage disequilibrium between disease predisposition loci. Four disease scenarios were considered that are representative of complex diseases, such as major depression, hypertension, heart disease, or type II diabetes; a population prevalence of 5% or 10%; and heritability on the observed disease scale of 10% or 20%. These choices of parameters translate to diseases with relative risks for full-sibs of affected probands (K) ranging from 1.45 to 2.90. The analytic formula for the number of loci, assuming all loci to have the same effect and the same allele frequency, was found to be a robust predictor of the number of loci estimated by simulation when frequencies and effect sizes were sampled from a distribution; using Equation 3 with allele frequency, p = 0.5 or 0.1, gave results that agreed well with the CDCV or neutral model simulation, respectively. The analytic result is a convenient way to investigate the impact of disease prevalence and heritability on the number of loci underlying complex disease (Fig. 3). The number of disease risk loci that underlie complex disease have previously been investigated (Yang et al. 2005) based on an epidemiological parameterization using population-attributable fractions (rather than heritability), assuming equal frequencies and effects of risk loci. In Appendix A we derive closed-form solutions for the number of disease risk loci based on their parameterization.

In addition to the simulations reported here, we also simulated a disease with prevalence K = 0.01 and heritability h^2 = 0.05 (corresponding to a sibling RR of 3.48) and found results similar to those for a disease with similar relative sibling RR, e.g., K = 0.05, h^2 = 0.25. As with the other disease scenarios, we were

![Image](https://www.genome.org)

Figure 2. Relationship between the number of susceptibility or risk loci and their average relative risk (RR) for common disease; K is the population prevalence of the disease; h^2 is the heritability on the observed scale; λs is the RR for full-siblings based on the heritability and prevalence parameters. Distribution of effects of risk loci under neutral (A) and CDCV (B) models. The mean RR are the mean of 10,000 simulated samples.
able to identify individuals who had an increased risk of disease that was three to five times higher than average, but when prevalence is so low this still translates to a small absolute risk of disease, and so genomic profiling may be less useful for rare diseases. However, we note that low-prevalence diseases often show evidence for nonadditive genetic effects (monozygotic twin concordance rates several fold higher than dizygotic twin concordance rates, e.g., schizophrenia, type 1 diabetes, Crohn’s disease), implying that models that include nonadditive genetic effects may be more relevant to these disorders.

Our results show that, even for diseases controlled by 1000 loci with mean RR of only 1.04, a case-control study with 10,000 cases and controls can lead to selection of ~75 loci that explain >50% of the genetic variance, resulting in accuracy of risk prediction of >0.75. Prediction of increased risk to disease will be more successful for diseases with a high sibling risk. However, for diseases of low prevalence, the increased RR may translate into only a small absolute risk to disease. In contrast, for a disease of higher prevalence, the increase in RR for the top proportion of a population based on estimated risk may be small but may translate into a substantial absolute risk of disease. Examination of the true RR of disease for the individuals with the highest 5% of predicted risk in a new sample relative to the mean empirical population risk (Fig. 5) shows that, even with a case-control sample of 1000 for generation of risk profile SNPs, individuals who have risk of disease three times higher than the population average can be identified when the number of disease loci is <50. When the case-control sample is 10,000, individuals in the population that have a three to seven times increased risk of disease can be easily identified even when the number of disease loci is very large (1000). That is, individuals that have an absolute risk of disease of 15%–70% can be identified for the prevalences we have considered. Therefore, following a single large case-control study and the selection of a small set of SNPs, 100,000s to millions of individuals could be identified subsequently that are at substantial increased risk of disease relative to the population average. Genomic profiles can be viewed as a more accurate predictor of genetic predisposition than family history. The accuracy of prediction that we report can be compared with that achieved from a family history consisting of the disease status of a person’s mother and father, which is only 0.2–0.3 (≈0.5 h²; Falconer and Mackay 1996) for the disease scenarios considered. Whereas all sibs within a nuclear family will have the same risk based on family history, a genomic profile will provide individual risk profiles utilizing information from inherited risk loci (after all, half of the genetic variation occurs within families), even if disease has not been expressed in family members.

The accuracy of prediction (r̂gg) is the correlation between true and predicted genetic risk. This measure is appropriate to evaluate the utility of genetic profiling because it is the precision with which we can predict genetic risk. Whether an individual

![Image](https://www.genome.org)

Figure 3. Relationship between disease prevalence (K) and heritability (h²) on number of risk loci contributing to a disease, assuming a fixed frequency of risk alleles (p) and fixed RR of 1.1 (Equation 3). Based on results from Figure 1, p = 0.1 approximates to the neutral model and p = 0.5 approximates to the CDCV model.

Table 1. Summary statistics for the risk allele models

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with known genetic risk goes on to develop the disease is dependent on known and unknown environmental risk factors. In practice, genomic profiling would be used in combination with information of known environmental risk factors (Lyssenko et al. 2005). Without any such information, the accuracy of predicting disease status is simply \( \hat{r}_{gg} \). Janssens et al. (2006) suggest that the ROC (receiver operator curve)-based measure of discriminative accuracy, the area under the curve (AUC), is an appropriate measure for evaluating efficiency of genetic profiling, although others have disagreed (Lyssenko et al. 2006). A predictor of genetic risk cannot do better than predict the true genetic risk with an accuracy of 100%. Yet even in this situation, Janssens et al. (2006) noted that AUC accuracy is a function of heritability and disease prevalence. Therefore, AUC seems to us to be a confusing statistic, and it is preferable to quote the accuracy of the prediction, \( \hat{r}_{gg} \).

To investigate the use of high-density genome-wide genetic markers for prediction of genetic risk of disease, we have made some simplifying assumptions. We assumed that the true causal SNPs were always included in the GWAS, and we ignored linkage disequilibrium (LD) between simulated SNPs. If all of our SNPs are viewed as "tag-SNPs" (Carlson et al. 2004) that are selected to tag ungenotyped SNPs with a minimum \( r^2 \) value of 0.8, then 500,000 carefully selected SNPs will capture nearly all of the common variation in the genome (Barrett and Cardon 2006). In this case, our accuracy of prediction may be less than that calculated in our study, by a factor of at most \( 0.9 \). We chose a simple method to select SNPs based upon a predetermined number of false positives (\( \sim 1 \) out of 500,000) and a stringent type I error rate of \( \frac{2 \times 10^{-5}}{60} \). More complex methods could be used to select SNPs and to ensure unbiased estimates of effect size of variants contributing to the genomic profile (e.g., Meuwissen et al. 2001; Tibshirani et al. 2002; Storey and Tibshirani 2003; Zollner and Pritchard 2007), and such methods could be adapted to account for LD between the SNPs. However, our demonstration that a simple method performs adequately suggests that improvements from applying other SNP selection algorithms are not going to change the overall conclusions. We also ignored dominance and epistasis in the genetic model for disease because there is overwhelming evidence that most genetic variation is additive by nature, even when genes interact at a mechanistic level (Barton and Keightley 2002). Evidence that highly associated variants act additively (with disease risks of associated variants combining multiplicatively) in the same way as we have modeled has been shown recently for age-related macular degeneration (Maller et al. 2006). Nonetheless, evidence from experimental organisms

### Table 2. Summary statistics for selected SNP set (mean of 100 simulation replicates)

<table>
<thead>
<tr>
<th>( h^2 )</th>
<th>( K )</th>
<th>No. of risk loci</th>
<th>No. of selected loci</th>
<th>PVE</th>
<th>No. of selected loci</th>
<th>PVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.05</td>
<td>10</td>
<td>6.1</td>
<td>5.6</td>
<td>CDCV</td>
<td>Neutral</td>
</tr>
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<td>9.8</td>
<td>8.6</td>
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<td>69</td>
</tr>
<tr>
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<td>0.05</td>
<td>100</td>
<td>8.8</td>
<td>8.3</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
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<td>300</td>
<td>4.7</td>
<td>5.5</td>
<td>15</td>
<td>2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.05</td>
<td>1000</td>
<td>1.9</td>
<td>2.0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
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<td>0.05</td>
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<td>6.6</td>
</tr>
<tr>
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<td>11.9</td>
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<td>76</td>
</tr>
<tr>
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<td>0.05</td>
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<td>12.8</td>
<td>11.9</td>
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<td>60</td>
</tr>
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<td>9.4</td>
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<td>7</td>
</tr>
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<td>6.1</td>
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</tr>
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<td>1000</td>
<td>2.0</td>
<td>2.3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The number of loci selected from the genome-wide association study to predict risk in a new population sample of individuals and the percentage of the genetic variance of log risk explained by the selected loci (PVE) are shown.

**Figure 4.** Accuracy of risk prediction of disease risk in a population sample using a set of predictive SNPs selected after a genome-wide association study of \( N \) each of cases and controls. A CDCV disease model is assumed with population prevalence (\( K \)) and heritability (\( h^2 \)) of the disease. Results for the neutral model were similar. Mean of 100 simulation replicates. The legend lists the data series in their order at 1000 risk loci.
suggests that gene × gene interactions are likely to be important in some complex traits (Mackay 2004). We also did not consider gene × environment interactions; just as knowledge of environmental risk factors can be included into association studies, so can they be included in prediction of genetic risk to disease. Although many specific models could be defined, we believe the results given here will apply in general terms to a wide range of model assumptions. The high accuracy of prediction of risk that we achieve is partly attributable to the large case-control study samples that we have assumed, but large sample sizes are recognized as necessary for GWAS, and samples of >1000 cases and 1000 controls are already being genotyped (http://www.ncbi.nlm.nih.gov/WGA/programs/GAIN/data/; http://www.wtccc.org.uk/), with even larger samples used for replication studies (Cox et al. 2007; Sladek et al. 2007).

In the past, lack of replication has been a recurring problem for genetic association studies, which must, in part at least, be attributable to lack of power resulting from small sample sizes. In contrast, GWAS and their subsequent replication studies are characterized by large study samples. Time will tell if nonreplication of results and identification of large numbers of false positives is a characteristic of large-scale GWAS. Nonreplication of results may remain a problem if there are, as yet, undetermined methodological problems in genotyping, subtle population stratification effects, or important gene × environment interaction effects. If such problems exist, then our predictions for genetic risk provide an upper bound on the potential for prediction of genetic risk.

GWAS for type 2 diabetes have just been published (Saxena et al. 2007; Scott et al. 2007; Zeggini et al. 2007), and so far the results have been similar. Mean of 100 simulation replicates. The legend lists the data series in their order at 1000 risk loci.

We considered two models for the distribution of risk effects, and we assumed that all genetic variance was attributable to variants of frequency 0.01 to 0.99. If the true genetic architecture underlying complex diseases means that the majority of genetic variance is explained by variants with minor allele frequency <0.01 (the rare variants model; Pritchard 2001), then GWAS will fail to detect risk variants that explain much of the genetic variance because of lack of power and because very rare variants will not be tagged in a set of 500,000 SNPs. If GWAS fail to detect risk variants, then prediction of genetic risk will also fail if it is solely based upon the results from the GWAS. If only part of the genetic variance is available for detection, \( h^2_{\text{available}} \), then accuracy of prediction will decrease by a factor of \( h^2_{\text{available}} / h^2 \).

Under the neutral model we have simulated –50% of the risk variants have minor allele frequency <0.10 (Fig. 1), and these variants explain –20% of the genetic variance (Table 1). The first results of the Wellcome Trust Case Control Consortium (2007) have been published and have shown that the number of risk loci that they detect, the proportion of variance they explain, and the conclusion that larger sample sizes are needed are all in line with the models we have used.

Our simulation model allows direct investigation of the most important underlying factors that drive whether genomic profiling is feasible. Our results provide a foundation stone upon which further layers of complexity can be added, but such an exercise is only worthwhile if the foundation is sufficiently solid. All the caveats that apply to GWAS and their replication apply to the derivation of a SNP set that together predict genetic risk and its validation, ensuring that discovery, validation, and application populations are the same.

The need for new methodology for prediction of genetic risk has been recognized (Collins et al. 2003; Bell 2004; Khoury et al. 2006). Implementation of risk prediction in a clinical context has serious ethical and social implications (Grosse and Khoury 2006; Khoury et al. 2006) but has the potential to be of major economic benefit to population health (Khoury et al. 2006). Our study shows that prediction of genetic risk is possible, even if there are hundreds of risk variants each of small effect. Following a large single-stage GWAS (probably larger than those that are currently taking place worldwide), a set of SNPs can be selected that can accurately predict risk of disease in the population. For our prediction, it does not matter (assuming no population stratification), as long as the selected SNPs are true positives, whether they are in linkage disequilibrium with causal variants or causal themselves. The value of these predictive SNPs could be reapplied long before the causal mechanism of each contributing variant can be determined.

**Methods**

The success of association studies and also of genomic profiling depends on the genetic architecture underlying complex diseases. Our first aim is to investigate the relationship between the RR of genetic loci and the number of loci that contribute to risk of a disease under constraints of known disease prevalence and heritability. Ultimately, we will model the genetic architecture of complex disease by allowing the effect size and frequency of risk allele to vary across loci. However, to give insight into our results we first derive an analytical expression for the number of loci that contribute to a disease when the RR (\( \lambda \)) and the allele fre-
frequency \((p)\) of the risk alleles are both fixed. We will go on to use these results to investigate the potential of using multiple risk loci identified in a genome-wide association study to predict risk of disease in a new population cohort.

We introduce the following notations:

\[
\begin{align*}
    n & = \text{number of risk loci} \\
p & = \text{frequency risk allele (A)} \\
1 - p & = \text{frequency of resistant or "wild-type" allele (a)} \\
f_0 & = \text{probability (affected) | wild-type alleles at all loci} \\
\lambda & = \text{relative risk of a risk allele} \\
h^2 & = \text{heritability of the disease on the observed scale} \\
K & = \text{disease prevalence in the population}
\end{align*}
\]

Number of loci underlying complex disease when frequency and RR of risk loci are fixed across all loci

To model the underlying genetic control of complex diseases, we build upon the disease model suggested by Risch (Risch 1990; Risch and Merikangas 1996). We assume that all genotypes are in Hardy–Weinberg equilibrium, so that the probabilities of wild-type, carrier, and homozygous risk genotypes are \((1 - p)^2, 2p(1 - p),\) and \(p^2\), respectively. The relative risk of the carrier and homozygous risk genotypes are assumed to be \(\lambda\) and \(\lambda^2\), respectively.

Let \(g = \text{Prob(affected | genotype)} = f_0\lambda^x\), where \(x\) is the total number of risk alleles across all loci. Since we assume Hardy–Weinberg equilibrium, \(x\) is distributed binomial \((2n, p)\). Given the population parameters \(K, p,\) and \(h^2\), we can derive the number of loci, using the mean and genetic variance of the probability of an individual being affected.

\[
K = E(g) = E(f_0\lambda^x) = f_0\left[\frac{x\lambda}{N} + (1 - p)\right]^{2n} = f_0\left[1 + p(\lambda - 1)^2\right]^{2n} \quad (1)
\]

\[
\text{var}(g) = \text{var}(f_0\lambda^x) = E\left[(f_0\lambda^x)^2\right] - \left(E[f_0\lambda^x]\right)^2 = f_0^2\left[\left(1 + p(\lambda^2 - 1)\right)^{2n} - \left(1 + p(\lambda - 1)^2\right)^{2n}\right] \quad (2)
\]

For \(n = 1\), the population mean and the genetic variance reduce to those derived by Risch (1990). The variance of disease prevalence due to genetic factors is \(h^2K(1 - K)\). Hence, using this expression and Equations 1 and 2, we can solve for \(n\). After some algebra,

\[
n = \frac{1}{2}\left\{\ln[h^2 + (1 - h^2)K] - \ln(K)\right\} / \left\{\ln(1 + p(\lambda^2 - 1)) - \ln(1 + p(\lambda - 1))^2\right\}. \quad (3)
\]

Number of loci underlying a complex disease when frequency and RR of risk alleles vary across loci

Next we investigate the number of loci underlying complex disease of given disease prevalence and heritability when the frequency and RR of risk alleles vary. In this situation there is no simple analytical method to derive the mean RR needed to explain the genetic variance of disease for a given number of loci. For \(n\) loci with allele frequency \(p_i\) and relative risk parameter \(\lambda_i\), the mean and variance of risk \((R)\) are defined as

\[
K = E(g) = f_0E\left[\prod_{i=1}^{n}\lambda_i\right] = f_0E(R)
\]

with \(x\) the number of susceptibility alleles \((0, 1, 2)\) at the \(i\)th locus.

\[
\left[ E\left(\prod_{i=1}^{n}\lambda_i^{x_i}\right) - E\left(\prod_{i=1}^{n}\lambda_i\right)^{x}\right] = f_0^2\left[E(R^2) - E^2(R)\right]
\]

and \(CV^2 = \frac{\text{var}(g)}{K^2} = \frac{[E(R^2) - E^2(R)]}{E^2(R)} = E(R^2)/E(R) - 1\), where \(CV\) is the coefficient of variation. If we make the following approximations:

\[
E\left(\prod_{i=1}^{n}\lambda_i\right) = \prod_{i=1}^{n}[1 + p_i(\lambda_i - 1)]^2
\]

and

\[
E\left(\prod_{i=1}^{n}\lambda_i^{x_i}\right) = \prod_{i=1}^{n}[1 + p_i(\lambda_i^{x_i} - 1)]^2,
\]

then the genetic variance can be calculated conditionally on the allele frequencies and relative risks, without the need to sample genotypes for multiple individuals. The approximation was checked by simulation and was found to work well. Two distributions of frequency of risk alleles were considered: A uniform distribution of allele frequencies that broadly corresponds to the common-disease common-variant (CDCV) hypothesis (Chakravarti et al. 1999; Reich and Lander 2001), in which the frequency of the increasing risk allele was simulated as \(p_i = \text{Uniform}(0.01, 0.99)\), or a U-shaped distribution that broadly corresponds the neutral allele hypothesis (Pritchard 2001). In this case, the density function of \(p\) is \(f(p) = C/P[1 - p]\), and the cumulative density function is \(F(p) = -C \ln(1 - p)p\), with \(C\) a constant. To force the cumulative density to 1, we integrate from 0 + δ to 1 − δ, with δ a small number. Solving \(F(1 - \delta) = -C \ln(1 - p)p\), and then solve for \(p\), as \(p = 1/[1 + \exp(-\delta + 1/2/C)]\). To avoid the simulation of many allele frequencies that are close to 0 or 1 (with resulting finite samples that would be monomorphic), we truncated the allele frequencies at 0.01 and 0.99. To truncate the allele frequencies at \(p_i = 1 - p_i\), satisfies the relationship, \(1/p_i = 1/\exp(1/2 - \delta/C)\), which can be solved iteratively. For \(p_i = 0.01, \delta = 0.069183\). For each of the \(n\) risk loci, RR was simulated as \(\lambda_i = 1 + x(\lambda_i - 1)\), with \(x\) - Exponential(1). This results in \(\lambda\) always being larger than 1.0, provided that \(\lambda_0\), an arbitrary input parameter is >1.0. The mean of the simulated RR is \(E(\lambda) = \lambda_0\). The \(\lambda_i\) are transformed so that all the genetic variance is explained by the \(n\) loci as \(\lambda^2 = \lambda^2 + (\lambda_i - 1)/(\lambda_0 - 1)\), and the adjustment factor \(c\) was found iteratively so that it satisfies

\[
CV^2 = \frac{\sum_{i=1}^{n}[1 + p_i(\lambda_i^{x_i} - 1)]^2}{\prod_{i=1}^{n}[1 + p_i(\lambda_i - 1)]^2} - 1 = \frac{h^2(1 - K)}{K},
\]

or

\[
\sum_{i=1}^{n}[\ln(1 + p_i(\lambda_i^{x_i} - 1))]^2 - \sum_{i=1}^{n}[\ln(1 + p_i(\lambda_i - 1))]^2 = \ln[1 + h^2(1 - K)/K].
\]

The mean RR of the \(n\) simulated loci is \(R^2 = \sum_{i=1}^{n}(\lambda_i^2/m)\), and the \(\lambda_i^2\) are distributed \(1 + x(\lambda^2 - 1)\) with \(x\) - Exponential(1). This procedure of simulating and transforming relative risks for a fixed number of loci was implemented to force the set of simulated risk alleles to be consistent with a given heritability and disease
prevalence, while keeping the distribution of the effects exponential.

Analysis of case-control data for identification of multiple risk loci

Using our models for the genetic architecture of complex diseases, we go on to investigate prediction of genetic risk to disease from multiple risk loci identified in a GWAS. To do this we simulated a case-control study assuming a single-stage genome-wide association screen with 500,000 SNPs. The number of risk loci was fixed at 10, 20, 50, 100, 300, or 1000, and allele frequencies were simulated from either the U-shaped (neutral) or uniform distribution (CDCV). RR of disease loci were simulated from an exponential distribution, forcing the mean RR to be consistent with the heritability, population prevalence, and sampled allele frequencies (as described above). For all other nonsusceptibility loci, the RR was 1.0 and the allele frequencies were sampled as described for the risk loci. Using the sampled allele frequencies, genotypes for each independent locus for each simulated individual resulted from two independent draws from a Bernoulli distribution, which implies Hardy–Weinberg equilibrium. Disease status was simulated from the genotypes at the true n susceptibility loci using a Bernoulli distribution with probability

\[ P(D_j|G_i) = \frac{1}{\exp^\beta + 1} \]

where \( x_{ij} \) is the number of susceptibility alleles for individual i at locus j (\( x_{ij} = 0, 1, 2 \)). A Newton–Raphson algorithm was applied, which converged in approximately four iterations. In the rare cases where the count for a genotype was zero, the count was set to 1/2. Loci were selected for subsequent prediction of risk if the test statistic (a \( \chi^2 \) test) for association was above a predetermined threshold of 22.59, which corresponds to an expected number of one false positive from 500,000 tests and a nominal P-value of 2 \times 10^{-6}.

Risk prediction in a new sample from the same population

Next we used the SNPs selected from the simulated case-control study to see how accurately they could predict risk of disease in a randomly identified population sample. Therefore, we simulated a new independent sample of multilocus genotypes of 1000 individuals using the same properties of the multiple simulated risk loci. We assumed that disease status was unknown at the time of predictive testing but that subsequently disease status was known. Disease status for an individual was simulated conditionally on the simulated genotype (G). For each of these individuals, we knew the true disease probability and estimated disease probability from the selected SNPs, calculated as

\[ P(D_j|G_i) = \frac{1}{\exp^\beta + 1} \text{ and } \hat{P}(D_j|G_i) = \frac{1}{\exp^\beta + 1} \]

with n the total number of true risk loci, m the number of selected loci (both true and false), \( \hat{\beta}_j \) the estimated RR for locus j from the case-control study, and \( x_{ij} \) the number of risk alleles for individual i at locus j. Note that the estimated risk will deviate from 1.0 only for the selected loci. Probabilities of disease based upon observed genotypes at the selected SNP loci were estimated for all 1000 individuals and were ranked to identify those individuals that were predicted to be at most risk. The number of people with disease (simulated from the genotypes at all true susceptibility loci) was counted among the top 5% of these ranked probabilities. The ratio of the disease risk among the top 5% relative to the disease risk in the entire sample was calculated. This is the observed risk of the identified 5% of people that are most at risk. The accuracy of prediction was quantified by calculating the correlation between the logarithms of the true and predicted probability of disease.

Disease parameters

We considered two disease prevalences, \( K = 0.05 \) or 0.10, and two heritabilities on the observed disease scale, \( h^2 = 0.1 \) or 0.2. A total of 100 replicates were simulated for each scenario. Note that we assume additive gene action on the log risk scale (multiplicative gene action on the risk scale), that loci act independently, and that there is no linkage disequilibrium between disease predisposition loci. Additive gene action on the log risk scale approximates to having additive action on an underlying liability scale (Lynch and Walsh 1998). The heritability and prevalence parameters that we used can be interpreted in terms of the relative risk of a full-sib of an affected individual \( \lambda = 1 + 0.5 h^2(1 - K)/K \) (Lynch and Walsh 1998). This approximation assumes additivity on the risk scale, whereas we have additivity on the log risk scale. For a fixed number of loci, the mean RR is proportional to \( h^2(1 - K)/K \), that is, proportional to \( 1 - \lambda \).

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Appendix A

Closed solutions for number of risk loci for parameterization of Yang et al. (2005).

Yang et al. (2005) investigated the number of genes underlying complex disease by using an epidemiological framework. They consider an additive-effects (on the risk scale) model for which disease prevalence (using our notation for their Equation 1) is

\[ K = f_0 \sum_{j=0}^{n} \frac{n!}{(n-j)!} p_j (1-p_j)^{n-j} [x_{ij} - (j-1)] \]

where \( p_j \) is the frequency of risk genotype at a risk locus and \( x_{ij} \) is the RR of the risk genotype. Using the expected value of the number of risk genotypes n in the population of \( p_g \), this equation reduces to a closed form of \( K = f_0 [1 + np_g (\lambda_g - 1)] \). Defining population-attributable fraction (PAF), as \( PAF = (K - f_0)/K \), a closed solution for the number of risk loci is

\[ n = \frac{PAF}{(1 - PAF) p_g (\lambda_g - 1)^2} \]
They also consider a multiplicative-effects model (on the risk scale, additive on the log risk scale); their Equation 2, in our notation, is

\[ K = \sum_{i=0}^{n} \frac{n!}{(n-i)!} p_i (1 - p_i)^{n-i} \]

which, using the expected value of \( \lambda_i = [1 + p_i(\lambda - 1)] \), with \( x = 1 \) with probability \( p_x \) and \( x = 0 \) with probability \( 1 - p_x \), reduces to a closed form of \( K = \sum_{i=0}^{n} \frac{n!}{(n-i)!} p_i (1 - p_i)^{n-i} \lambda_i^x \)

solving for \( n \) gives

\[ n = \frac{\ln(1/(1-PAF))}{\ln(1 + p_x(\lambda - 1))} \]

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Prediction of individual genetic risk to disease from genome-wide association studies

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